Econ 230B Spring 2017

## **FINAL EXAM: Solutions**

The average grade for the final exam is 45.82 (out of 60 points). The average grade including all assignments is 79.38. The distribution of course grades is:

4 A+, 4 A, 4 A-, 6 B+, 5 B, 1-B, 1 C+.

## True/False Questions: 30 points

Answer all 10 questions (3 pts each). Explain your answer fully, since all the credit is based on the explanation.

Only short answers provided here. Full detailed in the class notes and relevant references.

1. Disability insurance has small negative effects on labor supply because empirical evidence shows that rejected disability insurance applicants work very little.

**Solution:** True based on the famous paper by Bound (1989) and subsequently verified on a bigger scale by Von Waechter-Manchester-Song (2011). Caveat: it is possible that the process of applying to DI, which requires not working for a number of months, could reduce labor supply of rejected applicants. See the recent evidence by Autor et al. 2015 discussed in class. Even with these effects factored in however, the negative effect of DI on work remains relatively small.

2. The US social security system discourages labor supply of the elderly because a significant fraction of US workers stop working at age 62. This response is predicted by the standard life-cycle model.

**Solution:** First part is true: there is a spike in retirement hazards at age 62 in the US that is driven by the early retirement age available at age 62 (nothing else changes at age 62 and this spike at 62 did not exist when the early retirement age was 65). Second part is false, if individuals were fully rational and as actuarial adjustment is close to fair, we should not observe a retirement spike at age 62.

3. The 2013 top tax rate increase in the United States led to a surge in reported top incomes in 2012 implying that tax rates on the rich have high efficiency costs.

**Solution:** Empirical statement is true. We do observe a spike in 2012 top incomes (see Saez TPE 2017) due to retiming of income from 2013 to 2012 to escape the higher 2013

rates. The efficiency statement is in part false because large retiming does not necessarily imply that the long-term response will be large (indeed findings by Saez TPE 2017 suggest small longer term responses). For tax policies that last many years, the relevant response to judge efficiency costs is the long-term response.

4. Evidence from lottery winners show that there are substantial income effects on labor supply.

**Solution:** Empirical evidence does show negative effects of lottery winnings on labor supply. Imbens et al. AER 2001 for the United States and Cesarini et al. 2015 for Sweden present compelling evidence comparing winners and non-winners conditional on playing. However, the magnitude of income effects is pretty small: \$1 reduces earnings by about \$0.1 so the empirical effects are not "substantial".

5. If individuals with no earnings are considered as less deserving than average by society, then an EITC with negative marginal tax rates at the bottom of the income distribution would be optimal even in the traditional Mirrlees model of optimal taxation.

**Solution:** This is true. The optimal tax rate at the bottom in the Mirrlees model takes the form  $T'(0) = (g_0 - 1)/(g_0 - 1 + e_0)$  with  $e_0 > 0$  the elasticity of the fraction non-working wrt to 1 - T'(0) and  $g_0$  the social marginal welfare weight on non workers. If individuals with no earnings are considered as less deserving than average by society, then  $g_0 < 1$  and therefore T'(0) < 0. Note that this result does not require responses along the extensive margin as in Saez QJE'02 (with extensive margin responses, T'(0) < 0 can be obtained under weaker conditions: low income workers more deserving than average).

6. An rise in the ratio of aggregate wealth to national income always leads to a rise in the capital share of national income, which in turn, everything else equal, usually leads to an increase in income inequality.

**Solution:** First part of the statement is false: this depends on the elasticity of substitution  $\sigma$  between capital and labor in production; the capital share rises if and only if  $\sigma > 1$ . The second part of the statement is true: because capital income is more unequally distributed than labor income, everything else equal a rise in the capital share tends to increase income inequality.

7. If the average rate of return to capital in the economy is 4%, an annual tax on wealth at a rate  $\tau_1 = 1\%$  is strictly equivalent to an annual tax on the flow of capital income at rate

 $\tau_2 = 25\%.$ 

**Solution:** False. The taxes  $\tau_1$  and  $\tau_2$  are equivalent for taxpayers who earn a rate of return of 4% on their wealth, but in practice there is significant heterogeneity in rates of returns across the distribution, due in particular to differences in portfolio composition. For taxpayers who have a rate of return different than 4%, the two taxes are different.

8. Formulary apportionment for the corporate income tax removes incentives for firms to move capital to low-tax countries.

**Solution:** Generally speaking this is false: it depends on the apportionment factors used in the formula. If capital enters the formula then incentives to move capital to low tax countries remain. Sales-based apportionment removes any such incentive.

9. Evidence from random audit studies show that there is not a lot of tax evasion in rich countries, especially at the bottom and middle of the income distribution.

**Solution:** True. Danish random audit studies find very low rates of evasion (tax gap of about 2.5%; see Kleven et al. 2011). US random audit studies find low levels of tax evasion too. The rates are blown up by a factor of about in the IRS tax gap studies but this factor is essentially arbitrary. Random audit studies, however, are not very informative about evasion at the top of the distribution as they miss sophisticated forms of evasion through legal and financial foreign intermediaries.

10. Because the US runs a trade deficit, a destination-based corporate cash-flow tax (DBCFT) would generate more revenue than the current US corporate tax keeping the corporate tax rate unchanged.

**Solution:** This is true in the short-run, but uncertain in the long-run. In present value terms the net trade balance of the US cannot be negative; therefore at some point the US will have to run a trade surplus, which would reduce the revenue from the DBCFT... unless the US is able to actually run persistent trade deficits (due, e.g., to a persistent returns differentials between US foreign assets and liabilities).

## PROBLEM (30 pts):

## 2. Optimal Linear Income Taxation:

Consider the following linear income tax problem. Individual utility is given by u(c, l) where c is consumption and l is labor supply. u(c, l) naturally increases with c and decreases with l. Each individual has an exogenous wage rate w distributed with density f(w) in the population (normalized to one). The minimum w is zero in the population.

The government uses a linear tax with rate  $\tau$ . Tax revenue is redistributed as a uniform lumpsum grant R (no other government expenses).

Individuals choose labor supply l to maximize  $u(w \cdot l \cdot (1 - \tau) + R, l)$ .

Let us denote by  $l(w \cdot (1 - \tau), R)$  the Marshallian labor supply function. Let us denote by  $Z = \int w lf(w) dw$  aggregate earnings in the economy.

In what follows a subscript denotes a partial derivative.

a) (2 pts) Let us assume throughout this problem set that the uncompensated elasticity of labor supply is positive and that leisure is a normal good. Show that this implies that l increases with  $w \cdot (1 - \tau)$  and decreases with R.

**Solution:** By definition, the uncompensated elasticity is  $e^u = (w(1-\tau)/l)\partial l/\partial(w(1-\tau))$  so  $e^u > 0$  implies l increases with  $w(1-\tau)$ . If leisure is a normal good, then leisure increases with R which implies that labor supply decreases with R (income effect parameter  $\eta = w(1-\tau)\partial l/\partial R$  is negative or zero).

b) (3 pts) Show that the government budget constraint is  $R = \tau \cdot Z$  and that this defines R as an implicit function of  $\tau$ . Show that  $R(\tau = 0) = R(\tau = 1) = 0$  and that  $R(\tau) > 0$  when  $0 < \tau < 1$ . Plot R as a function of  $\tau$ .

**Solution:** Taxes collected are  $\tau Z$  and fund R hence  $R = \tau Z$ . From  $l(w(1-\tau), R)$  we have that Z is a function of  $1 - \tau$  and R. Hence  $R = \tau Z(1 - \tau, R)$  defines R implicitly.

 $R(\tau = 0) = 0 \cdot Z(1, R) = 0$ . If  $\tau = 1$ , then l = 0 (not worth working), and hence Z = 0 so that R = 0. If  $0 < \tau < 1$  then l > 0 for those with w > 0. Hence, Z > 0 and R > 0.  $R(\tau)$  is the inversely U-shaped Laffer curve.

c) (3 pts) Using the fact from b) that  $R = \tau \cdot Z$  is a function of  $\tau$ , show that Z is an *increasing* function of  $1 - \tau$ .

Solution: We have

$$Z(1-\tau) = \int l(w(1-\tau), \tau Z(1-\tau))wf(w)dw$$

Hence

$$Z'(1-\tau) = \int [wl_{w(1-\tau)} + (-Z + \tau Z'(1-\tau))l_R]wf(w)dw$$
$$[1 - \int \tau l_R wf(w)dw]Z'(1-\tau) = \int [wl_{w(1-\tau)} - Zl_R]wf(w)dw.$$

 $l_R < 0$  and  $l_{w(1-\tau)} > 0$  proves the result that  $Z'(1-\tau) > 0$ .

d) (2 pts) Who is the worst off individual in this economy? What is the labor supply and utility of the worst off individual?

**Solution:** The worst off individual has w = 0 and hence does not work l = 0 and has utility u(R, 0).

e) (4 pts) Suppose the government is Rawlsian, i.e., government wants to maximize the utility of the worst off individual. Show that this implies that the government wants to set  $\tau$  to maximize R. Find a formula for the optimal  $\tau$  as a function of the elasticity e of aggregate earnings Z with respect to  $1 - \tau$ .

**Solution:** Maximizing the welfare u(R, 0) of the worst off individual is equivalent to maximizing R. Hence, the government chooses  $\tau$  to maximize  $R = \tau Z(1-\tau)$ . The FOC in  $\tau$  is  $Z - \tau Z'(1-\tau) = 0$  which can be rewritten as  $\tau Z'/Z = 1$  or  $\tau/(1-\tau) \cdot e = 1$  where  $e = (1-\tau)Z'/Z$  is the elasticity. Hence, we have  $\tau = 1/(1+e)$ .

f) (4 pts) Suppose that the government is utilitarian and maximizes:

$$W = \int u(w \cdot l \cdot (1 - \tau) + R, l) f(w) dw$$

subject to  $R = \tau \cdot Z$ . Derive the first order condition of the government program with respect to  $\tau$ .

Show that the optimal  $\tau$  can be written as:

$$\frac{\tau}{1-\tau} = \frac{1-g}{e},\tag{1}$$

where  $g = \int w \cdot l \cdot u_c \cdot f(w) dw / (Z \cdot \int u_c \cdot f(w) dw).$ 

**Solution:** The government chooses  $\tau$  to maximize:

$$W = \int u(wl(1-\tau) + \tau Z(1-\tau), l)f(w)dw$$

Thanks to the envelope condition, l is optimized by the individual and hence can be ignored in the government FOC which can be written as:

$$0 = \int [-wl + Z - \tau Z'(1-\tau)]u_c f(w)dw,$$

which can be rewritten as:

$$\tau/(1-\tau)\cdot(1-\tau)Z'/Z\cdot\int u_cf(w)dw = \int u_cf(w)dw - \int w lu_cf(w)dw/Z$$

hence, using  $g = \int w l u_c f(w) dw / (Z \cdot \int u_c f(w) dw)$ , we have:

$$\frac{\tau}{1-\tau} = \frac{1-g}{e}$$

g) (3 pts) Assuming that u(c, l) = u(c) - v(l) with u(c) concave increasing and v(l) convex and increasing, show that  $u_c$  decreases with w (for any tax rate  $\tau$ ). Show that this implies that 0 < g < 1 in question f).

**Solution:**  $u_c = u'(c) = u'(w(1-\tau)l + R)$ . As *l* increases with  $w(1-\tau)$ , we have  $w(1-\tau)l + R$  increases with *w*, and hence (as *u'* decreases),  $u_c$  decreases with *w*. *wl* increases with *w*.

Hence, wl and  $u_c$  are negatively correlated, which implies that:  $g = E(wl \cdot u_c)/E(wl)E(u_c) < 1$ 

h) (3 pts) Assuming that u(c,l) = c - v(l) with v(l) convex and increasing. Show that there are no income effects in labor supply in this case. What is the optimal utilitarian  $\tau$  from question f) in that case? Explain the economic intuition.

**Solution:** In this case  $u_c = 1$  for all individuals and hence g = 1 and hence  $\tau = 0$ . With utility linear in consumption, everybody has the same marginal utility of consumption and hence there are no utilitarian benefits from redistributing from rich to poor. As taxes create efficiency costs (and no benefits), the optimal tax is zero.

i) (3 pts) Suppose the government wants to estimate e in this economy using data on wage rates  $w_i$  and labor supply  $l_i$  for a small survey of individuals. Suppose that the economy is indeed exactly defined as in this problem set (i.e., this is NOT a real world question). What labor supply parameter would a regression of  $\log l_i$  on  $\log w_i$  identify? Would this labor supply parameter be sufficient to provide an estimate of the elasticity e relevant for the optimal tax formula (1) from question f) above?

**Solution:** The relationship between  $\log l_i$  on  $\log w_i$  captures the uncompensated elasticity of labor supply (changing  $w \cdot (1 - \tau)$  but keeping R fixed). The elasticity e also includes income effects (as R adjusts upwards to an increase in  $\tau$  further reducing labor supply through income effects). Hence, e is larger than the uncompensated elasticity.

j) (3 pts) In the set-up of question i), let us now assume that the government is Rawlsian as in question e). Would a regression of  $\log l_i$  on  $\log w_i$  identify the elasticity e relevant for the Rawlsian tax formula obtained in question e)?

**Solution:** In the Rawlsian case however, e does not include income effects because at the Rawlsian optimum, the lumpsum R is maximized. Therefore, a small change in  $\tau$  does not have a first order effect on R and hence generates a pure compensated effect. In fact,  $e = \bar{e}^u$  where  $\bar{e}^u$  is the average of the individual uncompensated labor supply elasticities  $e^u$  (weighted by earnings). Hence, in the Rawlsian case, the analysis of the relationship between  $\log l_i$  on  $\log w_i$  captures the relevant elasticity e for tax policy.