Spring 2021

ECON 133 "Global Inequality and Growth" Midterm

1. **True False Statement/Questions (10 points; 2 points each)**
Explain your answer fully based on what was discussed in lectures and sections. All the credit is based on the explanation.

   (a) Countries at the world's technological frontiers have experienced consistently high productivity growth rates throughout history, sometimes maintaining close to 10% growth rates over the course of several decades.

   - FALSE: For countries with the highest income and output per capita at the world's technological frontier, there is no case where they could sustain high productivity growth over a number of decades. This could change in the future depending on newly developed technology
   - No countries at the technological frontier have grown more than 2% per year for a sustained period of time
   - The exception is in Western Europe after WWII when they were catching up after the devastation of the war

   (b) The capital share of national income α always rises with the capital/income ratio β.

   - FALSE: it depends on the value of the elasticity of substitution between capital and labor σ. Recall that $\alpha = a\beta^{\frac{\sigma-1}{\sigma}}$ in the CES case, and therefore
     - If $\sigma > 1$: α is an increasing function of β
     - If $\sigma < 1$: α is a decreasing function of β

   (c) The functional distribution of income bears no relationship with the distribution of individual incomes.

   - FALSE: The decomposition of national income across factors of production (capital and labor) affects the distribution of individuals incomes. For instance, everything else equal, a higher capital share is associated with more personal income inequality, because capital income tends to be more concentrated than labor income.

   (d) In the long-run, and assuming that capital gains are negligible, the capital stock of the economy is equal to s/g times its national income (where s is the economy's saving rate and g the growth rate of national income).

   - TRUE: When we are in a steady-state and there are no price effects, β is equal to s/g
   - This is true for any model of wealth accumulation, as long as there are no long-run capital gains.

   (e) The income of the bottom 50% in the United States is lower after taxes and transfers than before.

   - FALSE: The tax-and-transfer system in the US is progressive overall.
   - The tax system is approximately a flat tax while transfers progressive; the combination of the two increases the income of the bottom 50% and reduces inequality.

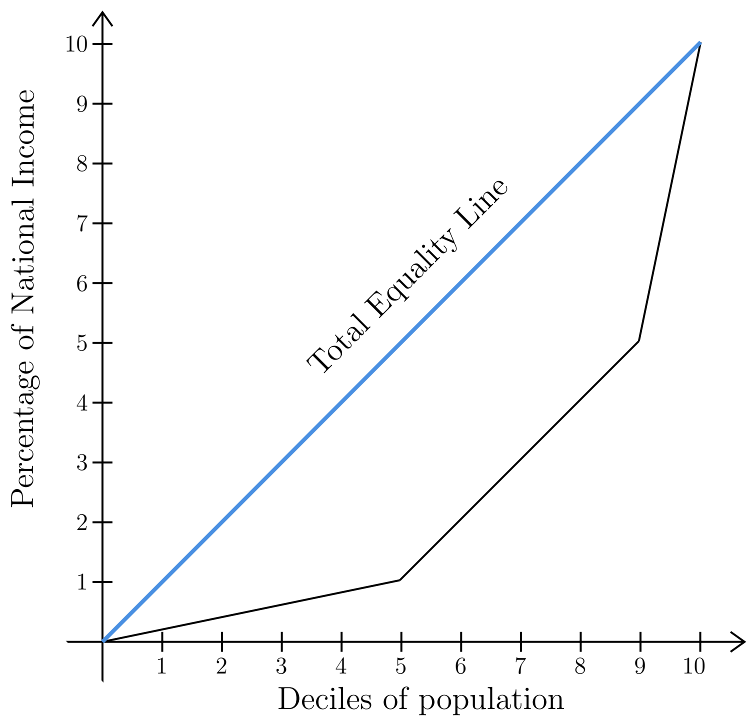
2. **Exercise 1 (14 points)**

Consider the case of an economy where national income is distributed as follows: the bottom 50% earns 10% of total income, the middle 40% earns 40%, and the top 10% earns 50%. Furthermore, assume that in each group everyone earns the same income (i.e., groups are homogeneous).

Part 1:

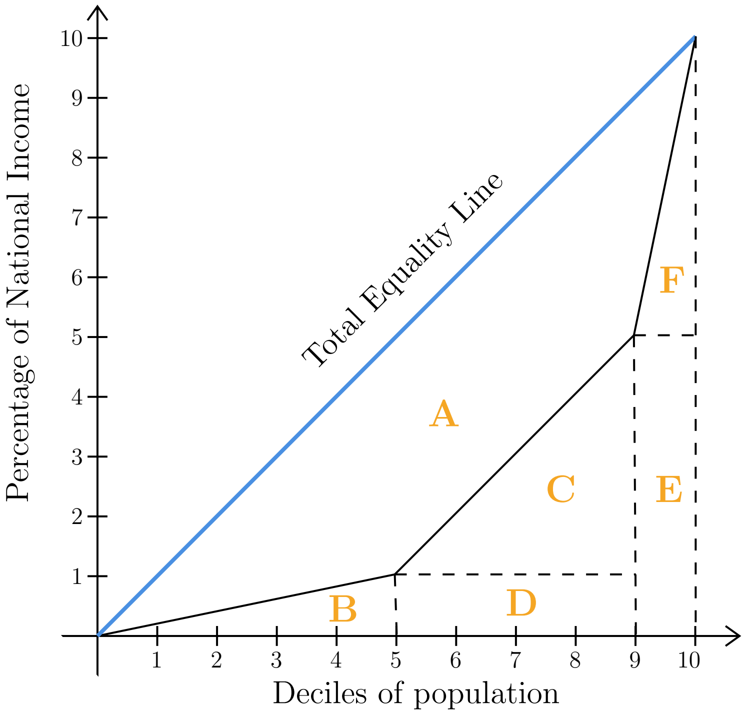
(a) Graph the line of total equality and the Lorenz curve. Clearly label each axis and data point (2 points).

Figure 1: Lorenz Curve: Homogeneous Groups



(b) What is the Gini coefficient in this economy? Clearly explain your computation (3 points).

Figure 2: Computing the Gini Coefficient



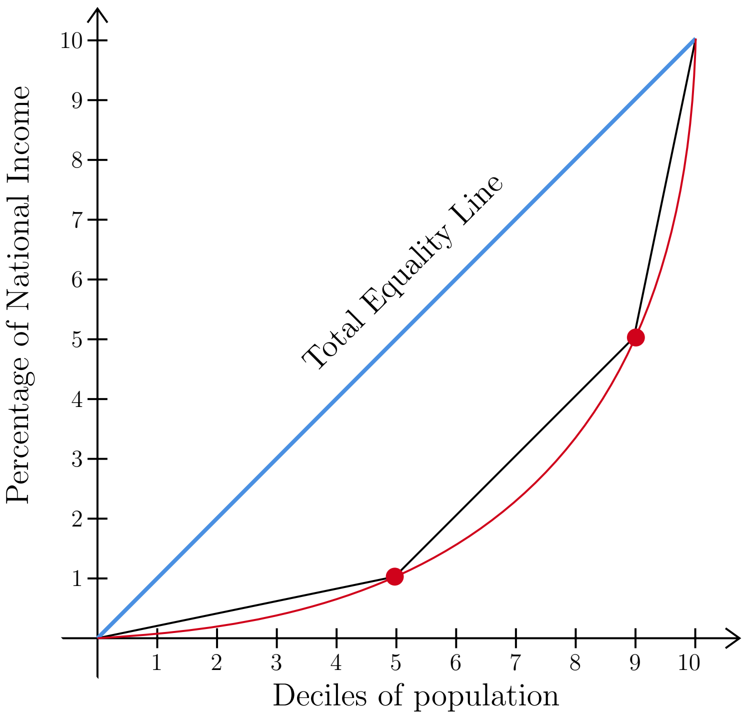
- Recall that the Gini coefficient is

$$
\begin{aligned}
G &= 2A \\
&= 2\left[\frac{1}{2} - B - C - D - E - F\right] \\
&= 2\left[\frac{1}{2} - \left(\frac{0.5*0.1}{2}\right) - \left(\frac{0.4*0.4}{2}\right) - (0.4*0.1) - (0.1*0.5) - \left(\frac{0.1*0.5}{2}\right)\right] \\
&= 2\left[\frac{1}{2} - \left(\frac{0.5*0.1}{2}\right) - \left(\frac{0.4*0.4}{2}\right) - 2\left(\frac{0.4*0.1}{2}\right) - 2\left(\frac{0.1*0.5}{2}\right) - \left(\frac{0.1*0.5}{2}\right)\right] \\
&= 1 - 0.5*0.1 - 0.4*0.4 - 2*0.4*0.1 - 2*0.1*0.5 - 0.1*0.5 \\
&= 1 - 0.05 - 0.16 - 0.08 - 0.1 - 0.05 \\
&= 0.56
\end{aligned}
$$

(c) Suppose now that there is inequality within groups, and each group's shares remain constant. Illustrate in the graph how the new Lorenz curve would look like. Is the Gini coefficient higher or lower? (1 point).

- The Gini coefficient is higher. There is more income dispersion since individuals within the same group do not earn the same.

Figure 3: Lorenz Curve: Heterogeneous Groups



(d) Assuming that you have all the information required to compute either coefficient, which measure do you prefer to quantify inequality: the Gini coefficient or top income shares (e.g., the share of income earned by the top 10%)? Why? This is an open question and credit fully depends on the coherence of the answer. (2 points)

- (This is an open question and credit fully depends on the coherence of the answer). It depends on the question you are interested in. The Gini coefficient measures inequality across the whole income distribution. Therefore, top income shares might miss an important margin. For example, if inequality is reduced within the bottom 50%, the group income shares remain the same and the change is not captured by the group income shares. However, the Gini coefficient is difficult to interpret in the void, and several income distributions are represented by the same Gini. On the other hand, top income shares are straightforward to interpret and capture inequality at the very top, which is usually what matters the most (because most of the income is concentrated at the top, and because inequality at the top might be normatively more problematic).

Part 2: Suppose that the average per capita income in this economy is 20,000. Remember that national income is distributed as follows: the bottom 50% earns 10%, the middle 40% earns 40%, and the top 10% earns 50%.

(e) What is the average income of each group? Clearly explain your computation. (2 points)

- Recall that

$$\textit{avg. income of group } i = \frac{\textit{income share of group } i}{\textit{population share of group } i} \times \text{avg. income of the country}$$

- Average income in the bottom 50%: $(0.1/0.5) \times 20,000 = 4,000$
- Average income in the middle 40%: $(0.4/0.4) \times 20,000 = 20,000$
- Average income in the top 10%: $(0.5/0.1) \times 20,000 = 100,000$

(f) Suppose that the top 10% can be characterized by a Pareto coefficient of 1.5. What is the inverted Pareto coefficient in this economy? (1 points)

- The inverted Pareto coefficient (or concentration coefficient) is $b = \frac{a}{a-1} = \frac{1.5}{1.5-1} = 3$.

(g) Assume that the threshold income to belong to the top 1% is 200,000. What is the income share of the top 1%? (Hint: you need to calculate the average income of the top 1%). (3 points)

- Since at the top 10% the income distribution follows a Pareto distribution, we know that the average income $y^*(y)$above a threshold y is the threshold times the inverted Pareto Coefficient.
- $y^*(200,000) = 200,000 \times b = 200,000 \times 3$.

⇒ The average income in the top 1% is 600,000.

- The income share of this group can be easily calculated using

$$\begin{aligned}
\textit{avg. income of group } i &= \frac{\textit{income share of group } i}{\textit{population share of group } i} \times \text{avg. income of the country} \\
600,000 &= \frac{\textit{income share of top 1\%}}{\textit{0.01}} \times 20{,}000 \\
\Rightarrow \textit{income share of top 1\%} &= \frac{600,000 \times 0.01}{20,000} = 30\%
\end{aligned}$$

- The income share of the top 1% is 30% of national income.

3. **Exercise 2 (6 points)**

Assume there are two countries that can be represented by the same CES production function

$$F(K,L) = \left(a \cdot K^{\frac{\sigma-1}{\sigma}} + (1-a) \cdot L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where $t = 1, 2$ and refers to country 1 and 2, and $\sigma = 1.5$. Suppose that in country 1, the capital income share (α_1) is 24%, and the wealth-to-income ratio (β_1) is 250%, and that in country 2 the wealth-to-income ratio is (β_2) is 800%.

(a) What is the capital share in country 2 (α_2)? (Hint: You need to use the formula covered in section, relating α, β and σ in the CES case) (3 points).

- We have seen in lecture and section that if the production function is CES, the capital share can be represented in terms of β and σ, according to the following formula: $\alpha_t = a\beta_t^{\frac{\sigma-1}{\sigma}}$. Therefore,

$$\alpha_1 = a\beta_1^{\frac{\sigma-1}{\sigma}}$$
$$\alpha_2 = a\beta_2^{\frac{\sigma-1}{\sigma}}$$

Taking the ratio of the above 2 expressions we get:

$$\frac{\alpha_2}{\alpha_1} = \frac{\beta_2}{\beta_1}^{\frac{\sigma-1}{\sigma}}$$
$$\leftrightarrow \alpha_2 = \frac{\beta_2}{\beta_1}^{\frac{\sigma-1}{\sigma}} \times \alpha_1$$
$$\leftrightarrow \alpha_2 = \left(\frac{800\%}{250\%}\right)^{\frac{0.5}{1.5}} \times 24\% = 35\%$$

(b) What would be the capital income share of country 2 if the production function of both countries was Cobb-Douglas instead? (1 point).

- We know from lecture that when $\sigma = 1$, the CES is Cobb-Douglas. Therefore,

$$\alpha_2 = \left(\frac{800\%}{250\%}\right)^{\frac{1-1}{1}} \times 24\% = 24\%$$

Actually, to derive this result we do not need any math. We know that in the Cobb-Douglas case, the capital income share is fixed, regardless of K and L. So, if both countries have the same technology, they also have the same capital income share (and thus $\alpha_1 = \alpha_2 = 24\%$).

(c) Assume we are in steady-state and that there are no capital gains. If the two countries have the same growth rate ($\bar{g}$), which one has the highest savings rate? How do you know? (1 point).

- Under these assumptions, we know that $\beta = \frac{s}{g}$ (HDS formula). Since $\beta_1 = \frac{s_1}{\bar{g}} = 250\%$ and $\beta_2 = \frac{s_2}{\bar{g}} = 800\%$, it is direct that $s_2 > s_1$.

(d) Derive the savings rate of country 1 (s_1) in terms of country's 2 savings rate (s_2) (1 point).

- Using the formula above,

$$\begin{aligned}
\beta_1 &= \frac{s_1}{\bar{g}} = 250 \Rightarrow s_1 = 250 \times \bar{g} \\
\beta_2 &= \frac{s_2}{\bar{g}} = 800 \Rightarrow s_2 = 800 \times \bar{g} \\
\Rightarrow s_1 &= 250 \times \frac{s_2}{800} \\
&= \frac{5}{16} s_2
\end{aligned}$$

4. **Bonus (2 points)**

- From the press articles and discussions circulated, briefly describe the characteristics of global wealth convergence during the COVID pandemic (verbatim or quoted text from the article/podcast will get no credit).
  - Global weatlh convergence refers to the decrease in global wealth inequality observed during the COVID-19 pandemic.
  - The author highlights how this convergence is a result of a more severe impact of the pandemic in higher-income economies than lower-incomes ones.
  - This implied a decrease in wealth for the rich, but not an improvement for the poor.