Econ 230B – Graduate Public Economics

Models of the wealth distribution

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Roadmap

1. The facts to explain

2. Precautionary saving models

3. Dynamic random shock models

1 The facts to explain

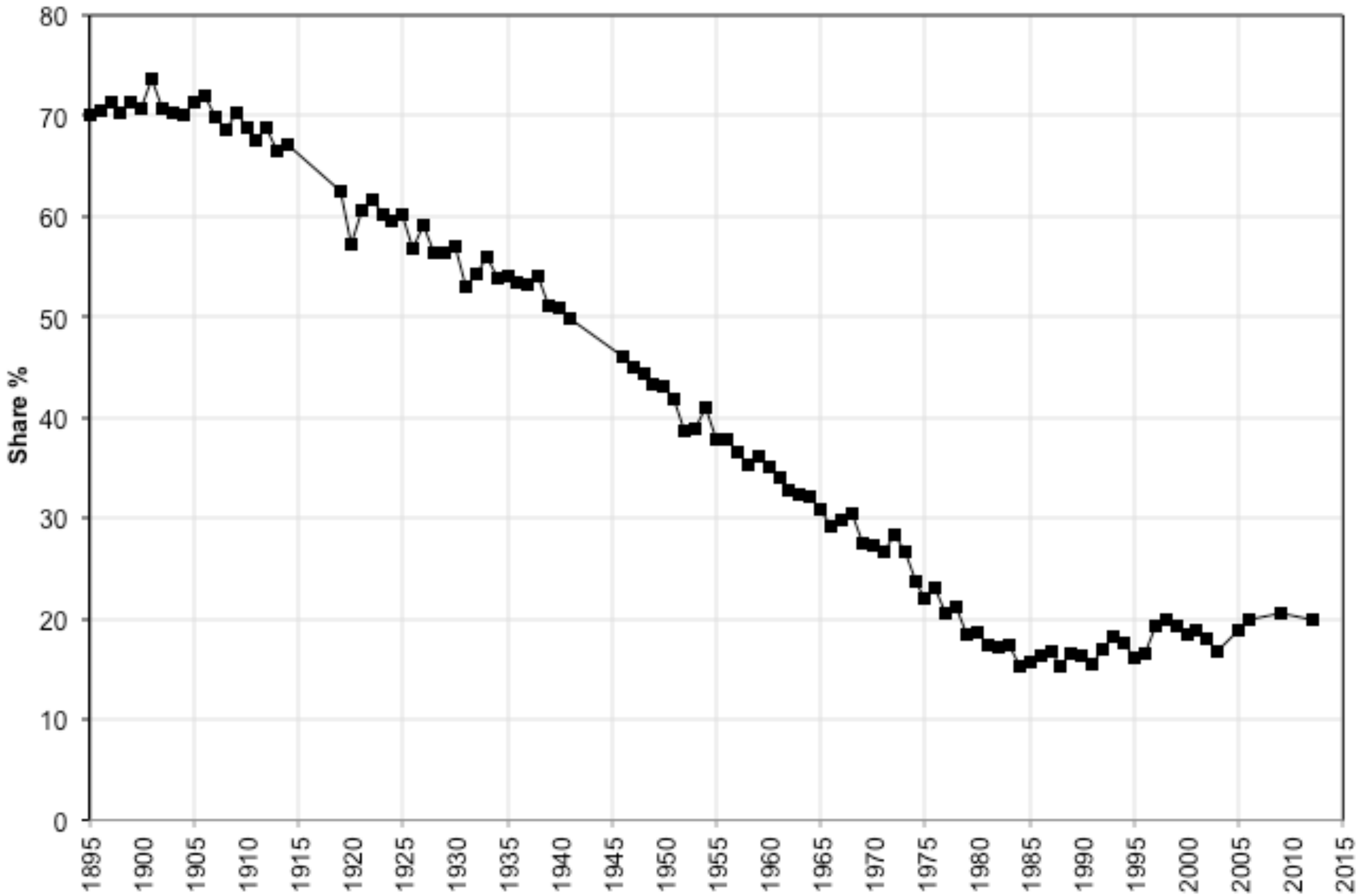
- Fact 1: Wealth is very unequally distributed, much more than labor income

- Fact 2: Wealth concentration tends to be particularly high in low-growth societies (e.g., 18th-19th century)

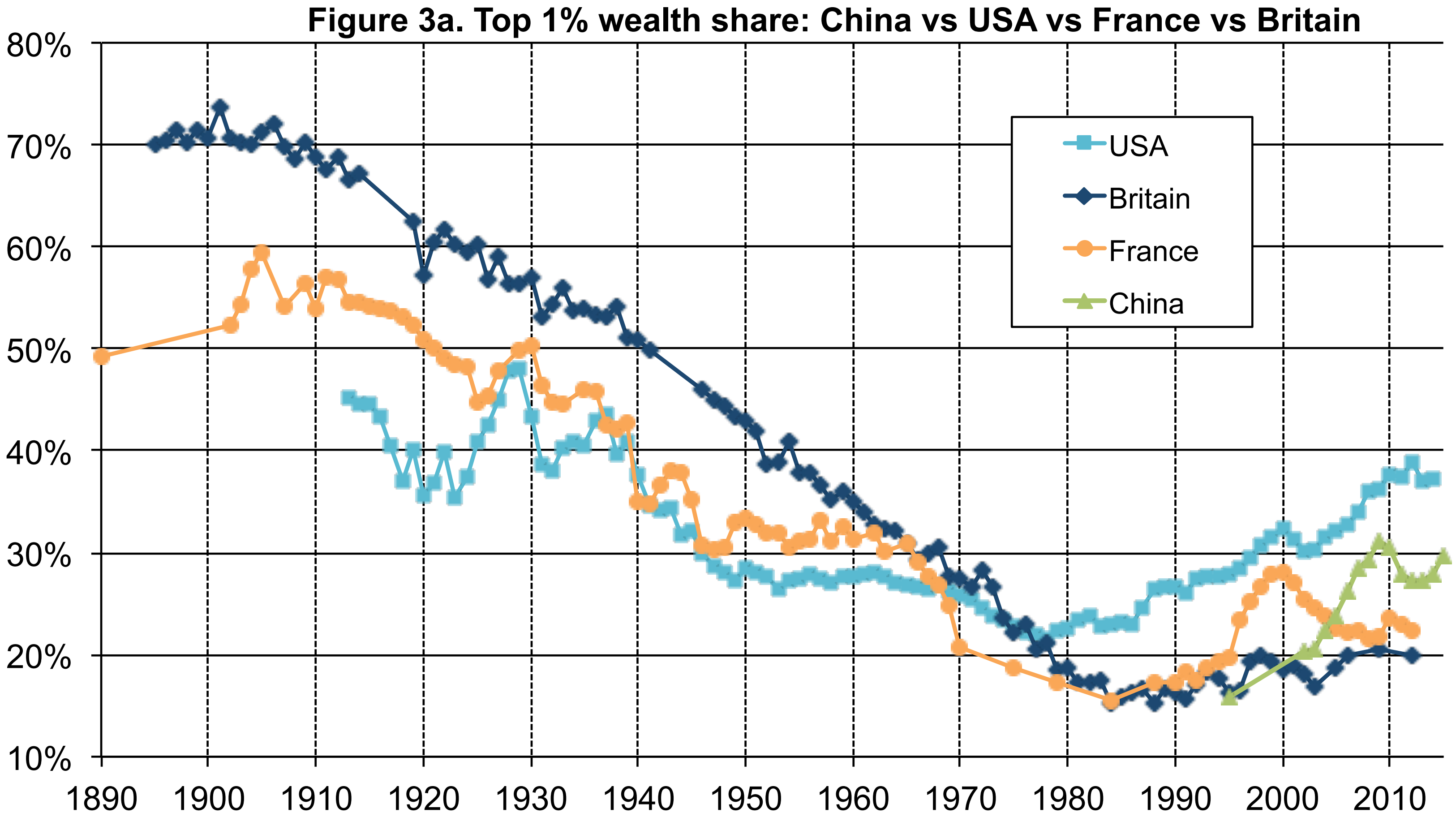
- Fact 3: Wealth inequality has been rising in recent decades but there is a diversity of national trajectories

Source: Piketty, Saez and Zucman (2016).

Figure 1. Wealth share of top 1% in the UK 1895-2013



Source: Alvaredo, Atkinson and Morelli (2017).



Distribution of net personal wealth among adults. Corrected estimates (combining survey, fiscal, wealth and national accounts data). Equal-split-adults series (wealth of married couples divided by two). USA: Saez and Zucman (2016). Britain: Alvaredo, Atkinson and Morelli (2017). France: Garbinti, Goupille and Piketty (2016). China: Piketty, Yang and Zucman (2016).

Source: Alvaredo et al. (2017).

2 Precautionary saving models

- General equilibrium models of wealth accumulation with non-insurable idiosyncratic risks

- Main form of risk: unemployment risk

- Other form of risk: fluctuation in earnings

- Widely used in macro to study the distribution of wealth and the effect of tax policies (see DeNardi & Fella 2017 for a survey)

2.1 Aiyagari (QJE 1994)

- Neoclassical growth model with a continuum of infinitely-lived, ex-ante identifical agents who max $U(c_0, c_1, ...) = E_0 \sum \beta^t u(c_t)$

- Idiosyncratic uninsurable shocks to endowment of efficiency units of labor follow Markov process $\pi(\epsilon', \epsilon) = Pr(\epsilon_{t+1} = \epsilon' | \epsilon_t = \epsilon)$

- Problem of each agent can be written in recursive form:

$$v(w, \epsilon) = \max_{c, w'} \left(u(c) + \beta \sum \pi(\epsilon', \epsilon) v(w', \epsilon') \right)$$

$$c + w' = (1 + r)w + v\epsilon \quad \text{and} \quad w' \geqslant -b$$

- Result 1: there exists a stationary equilibrium where the distribution of wealth is invariant and ergodic

- This is in contrast to a perfect market world (standard dynastic model) where any initial distribution of wealth is sustained forever

- Result 2: In contrast to Chamley-Judd, > 0 optimal capital taxation in such models (people save too much) (Aiyagari, JPE'95)

- Result 3: such a model does not generate much wealth inequality...

- Unless one chooses a sufficiently unrealistic income process (Castañeda et al., JPE'03). Even then, wealth not Pareto distrib.

3 Dynamic random shock models

- Consider dynamic equation for wealth z_i of the form

$$z_{t+1i} = \gamma_{ti} \cdot z_{ti} + \varepsilon_{ti}$$

- Where γ_{ti} are i.i.d. shocks with mean $0 < \gamma = E(\gamma_{ti}) < 1$

- ε_{ti} is a positive additive shock (possibly random)

- Then under a number of regularity assumptions, three key results:

  – The distribution of z_i converges to a steady state

– The steady-state distribution has a Pareto upper tail

– The Pareto coefficient a solves the following equation:

$$E(\gamma_{ti}^{a}) = 1$$

- The latest result was first shown by Champernowne (1953)

- The general study of these stochastic processes was rigorously done by Kesten (1973). See Gabaix (2009).

- Key intuition: cumulative multiplicative shocks lead to Pareto laws, but needs reflective barrier ε_{ti} to prevent process from diverging

Piketty-Zucman (HID 2015): Setup

- Discrete time $t = 0, 1, 2, ...$ (can be interpreted as one year or one generation)

- Stationary population $N_t = [0, 1]$ made of a continuum of agents of size one

- Aggregate and average variables are the same for wealth and national income: $W_t = w_t$ and $Y_t = y_t$

- Effective labor input $L_t = N_t \cdot h_t = N_0 \cdot (1 + g)^t$ grows at exogenous rate g

- Domestic output given by production function $Y_{dt} = F(K_t, L_t)$.

- Each individual $i \in [0, 1]$ receives **same** labor income $y_{Lti} = y_{Lt}$ and has same rate of return $r_{ti} = r_t$

- End-of-period wealth in utility function (flexible: middle-ground between life-cycle and dynastic model)

$$V(c_{ti}, w_{t+1i}) = c_{ti}^{1-s_{ti}} w_{t+1i}^{s_{ti}}$$

- Where s_{ti} is wealth (or bequest) taste parameter

- Budget constraint: $c_{ti} + w_{t+1i} \leq y_{Lt} + (1 + r_t) \cdot w_{ti}$

- Random shocks = idiosyncratic variations in saving taste s_{ti} drawn from i.i.d. random process with mean $0 < s = E(s_{ti}) < 1$

- Cobb-Douglas utility implies consumption c_{ti} is a fraction $1 - s_{ti}$ of $y_{Lt} + (1 + r_t) \cdot w_{ti}$, the total resources (income+wealth) available

- Plugging this formula into the budget constraint yields following individual-level transition equation for wealth:

$$w_{t+1i} = s_{ti} \cdot [y_{Lt} + (1 + r_t) \cdot w_{ti}] \tag{1}$$

Piketty-Zucman (2015): aggregate convergence

- At aggregate level, national income equals $y_t = y_{Lt} + r_t \cdot w_t$, hence

$$w_{t+1} = s \cdot [y_{Lt} + (1 + r_t) \cdot w_t] = s \cdot [y_t + w_t] \qquad (2)$$

- Divide by $y_{t+1} \approx (1 + g) \cdot y_t$, denote $\alpha_t = r_t \cdot \beta_t$ the capital share, $(1 - \alpha_t) = y_{Lt}/y_t$ the labor share to obtain transition equation for the wealth-income ratio $\beta_t = w_t/y_t$

$$\beta_{t+1} = s \cdot \frac{1 - \alpha_t}{1 + g} + s \cdot \frac{1 + r_t}{1 + g} \cdot \beta_t = \frac{s}{1 + g} \cdot (1 + \beta_t) \qquad (3)$$

- Solution to this dynamic equation? Two cases

- Open-economy case: world rate of return $r_t = r$ is given. β_t converges towards a finite limit β if and only if

$$\omega = s \cdot \frac{1+r}{1+g} < 1$$

- If $\omega > 1$, then $\beta_t \to \infty$. In the long run, the economy is no longer small, and world rate of return has to fall so that $\omega < 1$

- Closed-economy case: β_t always converges towards a finite limit because r adjusts (falls with β)

- Example: with a CES production function: $r = F_K = a \cdot \beta^{-1/\sigma}$

- Setting $\beta_{t+1} = \beta_t$ in equation 3, we have:

$$\beta_t \to \beta = s/(g+1-s) = \widetilde{s}/g$$

- where $\widetilde{s} = s(1+\beta) - \beta$ is the steady-state saving rate expressed as a fraction of national income

- See Piketty & Zucman (2014 QJE) for models of β in the long-run (whatever the utility function, $\beta \to s/g$)

- So macro variables converge to a steady-state, what about the distribution of wealth?

Piketty-Zucman (2015): convergence of wealth distribution

- Denote $z_{ti} = w_{ti}/w_t$ normalized individual wealth, and divide both sides of equation 1 by $w_{t+1} \approx (1+g) \cdot w_t$

- In the long run the individual-level transition equation for normalized wealth can be written as follows:

$$z_{t+1i} = \frac{s_{ti}}{s} \cdot [(1-\omega) + \omega \cdot z_{ti}] \qquad (4)$$

- (To see this, note that $y_{Lt} = (1-\alpha) \cdot y_t$, where $\alpha = r \cdot \beta = r \cdot s/(1+g-s)$ is the long-run capital share.)

Now apply Kesten (1973) theorem:

- Distribution $\psi_t(z)$ of relative wealth converges towards a unique steady-state distribution $\psi(z)$
- $\psi(z)$ has a Pareto upper tail
- Pareto exponent a is such that $E\left(\left(\frac{s_{ti}}{s} \cdot \omega\right)^a\right) = 1$

Example: binomial taste shocks

- $s_{ti} = s_0 = 0$ with probability $1 - p$ (consumption lovers)

- $s_{ti} = s_1 > 0$ with probability p (wealth lovers)

- Average saving taste $s = E(s_{ti}) = p \cdot s_1$

- If $s_{ti} = s_0 = 0$, then $z_{t+1i} = 0$: children with consumption-loving parents receive no bequests

- If $s_{ti} = s_1$, then $z_{t+1i} = \frac{s_1}{s} \cdot [(1 - \omega) + \omega \cdot z_{ti}]$: children with wealth-loving parents receive positive bequest growing at rate ω / p

By Kesten's (1973) theorem, $E\left(\left(\frac{s_{ti}}{s}\cdot\omega\right)^{a}\right) = (\omega/p)^{a}\cdot p = 1$, hence

$$a = \frac{\log(1/p)}{\log(\omega/p)} \tag{5}$$

$$b = \frac{a}{a-1} = \frac{\log(1/p)}{\log(1/\omega)}$$

- As $\omega = s\cdot(1+r)/(1+g)$ rises, Pareto coefficient a declines and inverted Pareto-Lorenz coefficient b rises: more inequality

- High ω means the multiplicative wealth inequality effect is large compared to the equalizing labor income effect

- In the extreme case where $\omega \to 1^-$ (for given $p < \omega$), $a \to 1^+$ and $b \to +\infty$ (infinite inequality)

- The same occurs as $p \to 0^+$ (for given $\omega > p$): an infinitely small group gets infinitely large random shocks

- Extreme concentration can also occur if taste parameter s_{ti} is higher on average for individuals with high initial wealth

- All models with multiplicative random shocks in the wealth accumulation process yield distributions with Pareto upper tails

- True whether shocks come from tastes or other factors

Stiglitz (Econometrica 1969)

- Shock is the rank of birth: primogeniture

- Generational growth g only comes from population growth n

- Each family has $1 + n$ boys, $1 + n$ girls

- Probability to be first-born son (= good shock) $p = 1/(1 + n)$

- Plug this into eq. 5 for a in binomial random shock model:

$$a = \frac{log(1 + n)}{log(s(1 + r))}$$

Cowell (1998)

- Shock is the number of children

- This is more complicated because families with many children do not return to zero wealth (unless infinite number of children)

- No closed-formed solution for a which must solve:

$$\sum \frac{p_k \cdot k}{2} \left(\frac{2 \cdot \omega}{k} \right)^a = 1$$

- $p_k =$ fraction of parents who have k children ($k = 1, 2, 3...$), $\omega =$ average generational rate of wealth reproduction

Benhabib, Bisin and Zhu (Econometrica 2011)

- Shocks come from rates of return $\rightarrow$ same Kesten multiplicative random shock process $z_{t+1i} = \gamma_{ti} \cdot z_{ti} + \varepsilon_{ti}$ as with random saving

- Rich set up: finite life with inter-generational linkages; endogenous saving; capital income taxes vs. wealth taxes...

- Allow for correlation between γ_{ti} (persistence in rates of returns across generations) and γ_{ti} and ε_{ti} (high labor income earners can have high rates of returns)

- Capital taxes reduce inequality a lot

Calibration of random saving taste model: the role of $r - g$

- Interpret each period as lasting H years (with $H = 30$ years = generation length)

- Let r and g denote instantaneous rates, then $1 + R = e^{rH}$ = generational rate of return; $1 + G = e^{gH}$= generatl. growth rate

- Multiplicative factor ω can be rewritten

$$\omega = s \cdot \frac{1 + R}{1 + G} = s \cdot e^{(r-g)H}$$

- If $r - g$ rises from $r - g = 2\%$ to $r - g = 3\%$, then with $s = 20\%$

and $H = 30$ years, $\omega =$ rises from $\omega = 0.36$ to $\omega = 0.49$.

- For a given binomial shock structure $p = 10\%$, the resulting inverted Pareto coefficien from $b = 2.28$ to $b = 3.25$.

- This corresponds to a shift from moderate wealth inequality (top 1% wealth share around 20-30%) to very high wealth inequality (top 1% wealth share around 50-60%).

- Small changes in $r - g$ can make a huge difference for long-run wealth inequality

Intuition: why $r - g$ matters

- $r - g$ magnifies any initial wealth inequality

- Ex: if $g = 1$ and $r = 4\%$, then a person whose income only derives from wealth W (hence has income rW) needs to save only g/r=25% for her wealth to grow as fast as the economy

- With taxes in the model, r must be replaced by the after-tax rate of return $\overline{r} = (1 - \tau) \cdot r$

- Where τ is the equivalent comprehensive tax rate on capital income, including all taxes on both flows and stocks.

Level and changes in $r - g$ gap can contribute to explain:

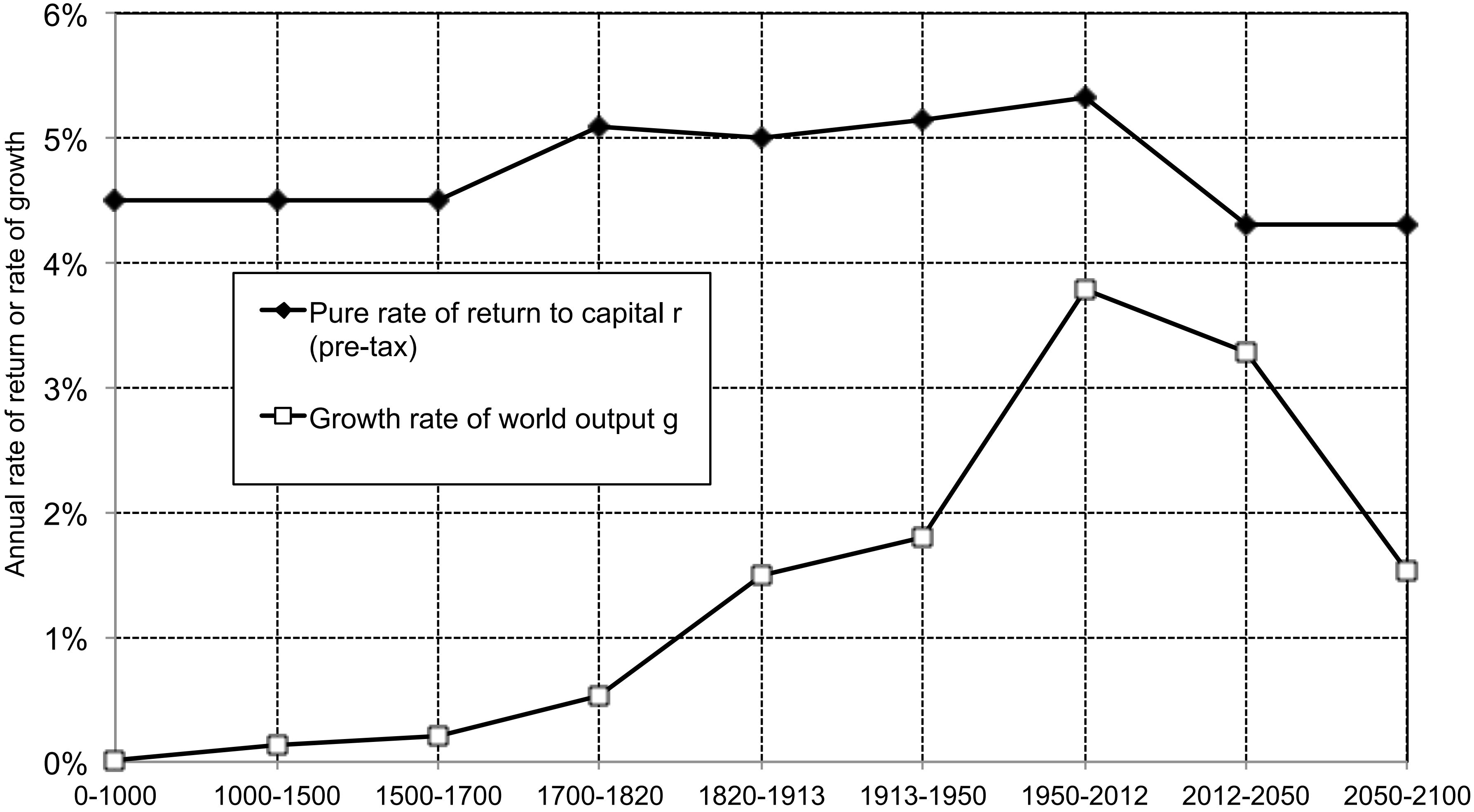
- Extreme wealth concentration in Europe in 19c and during most of human history (high $r - g$)

- Lower wealth inequality in the US in 19c (high g)

- Long-lasting decline of wealth concentration in 20c (low r due to shocks, high g)

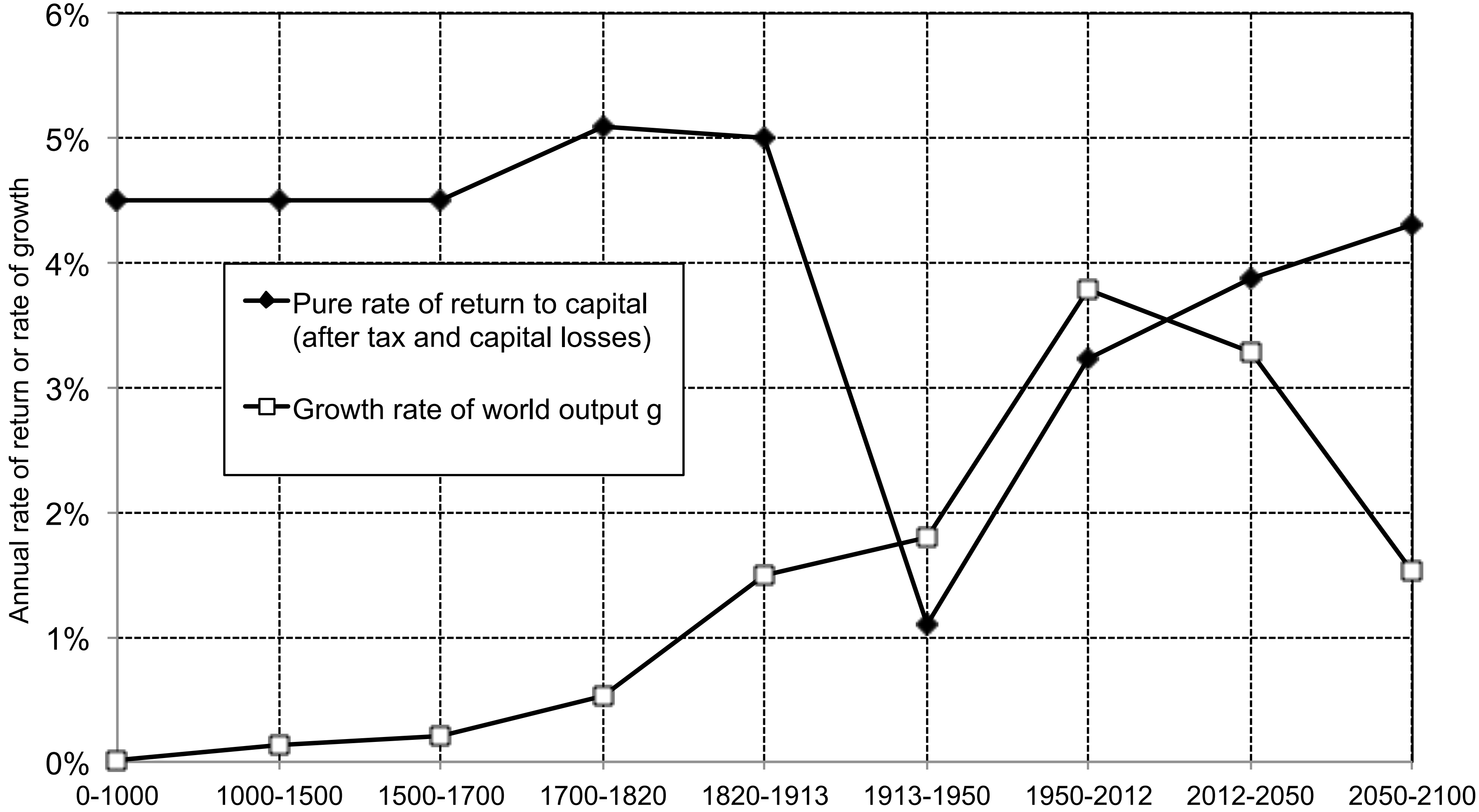
- Return of high wealth concentration since late 20c/early 21c (lowering of g, and rise of r, in particular due to tax competition)

Figure 10.9. Rate of return vs. growth rate at the world level, from Antiquity until 2100



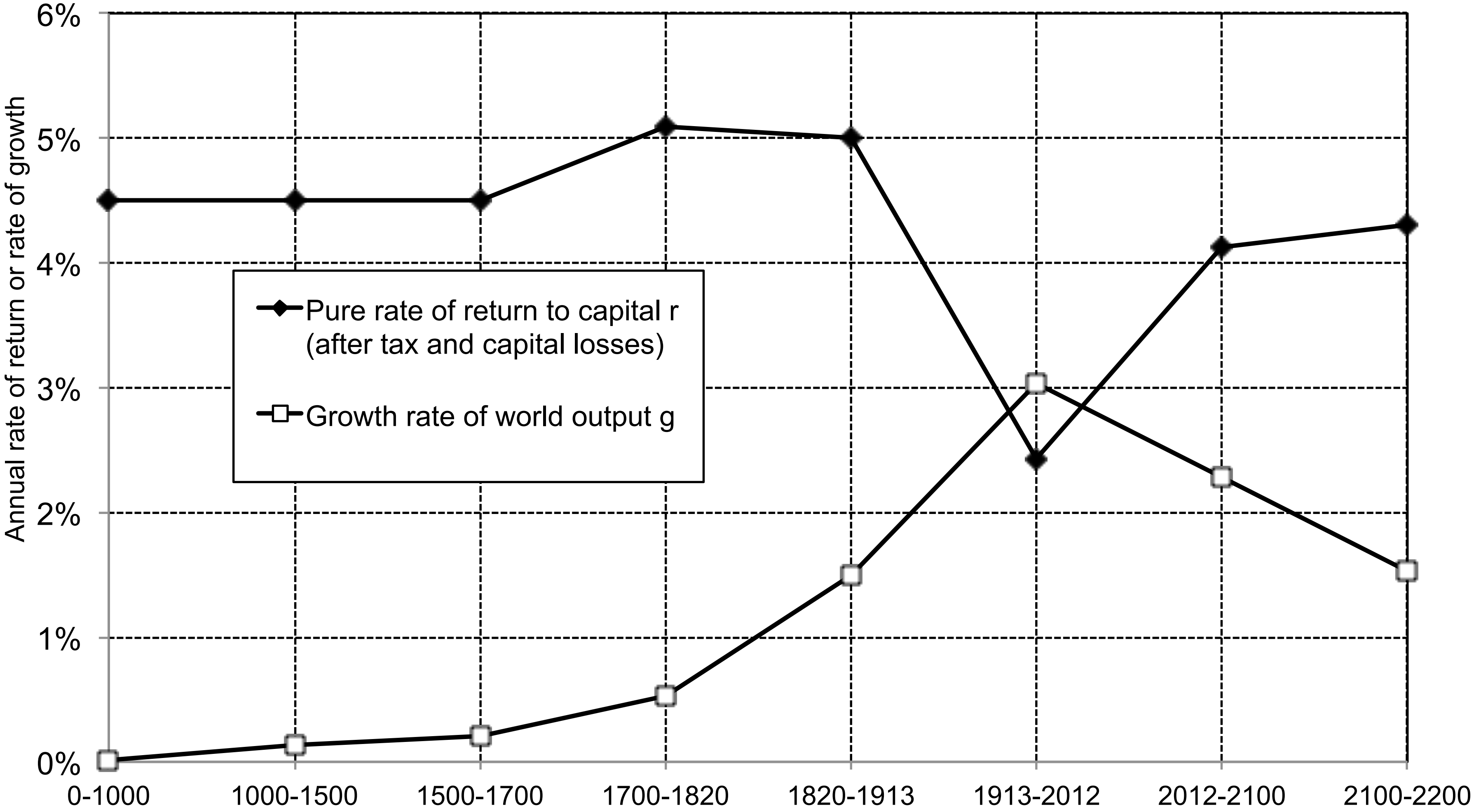
The rate of return to capital (pre-tax) has always been higher than the world growth rate, but the gap was reduced during the 20th century, and might widen again in the 21st century.
Sources and series: see piketty.pse.ens.fr/capital21c

Figure 10.10. After tax rate of return vs. growth rate at the world level, from Antiquity until 2100



The rate of return to capital (after tax and capital losses) fell below the growth rate during the 20th century, and may again surpass it in the 21st century. Sources and series : see piketty.pse.ens.fr/capital21c

Figure 10.11. After tax rate of return vs. growth rate at the world level, from Antiquity until 2200



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References

Aiyagari S.R. “Uninsured Idiosyncratic Risk and Aggregate Saving”, *Quarterly Journal of Economics* 1994 (web)

Aiyagari S.R. “Optimal Capital Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting”, *Journal of Political Economy* 1995 (web)

Alvaredo, Facundo, Anthony B. Atkinson, and Salvatore Morelli, “Top wealth shares in the UK over more than a century, ” CEPR discussion paper, 2017. (web)

Alvaredo, Facundo, Lucas Chancel, Thomas Piketty, Emmanuel Saez and Gabriel Zucman, “World Inequality Dynamics: New Findings from WID.world”, *American Economic Review P&P* May 2017 (web)

Benhabib, J., Bisin, A. and Zhu, S. (2011). “The Distribution of Wealth and Fiscal Policy in Economies with Finitely-Lived Agents”, *Econometrica* 2011 (web)

Castañeda A., J. Diaz-Gimenez and J.V. Rios-Rull, “Accounting for the US Earnings and Wealth Inequality”, *Journal of Political Economy* 2003 (web)

Champernowne D.G. “A Model of Income Distribution”, *Economic Journal* 1953 (web)

Cowell F., “Inheritance and the Distribution of Wealth”, working paper 1998 (web)

De Nardi, M. and G. Fella, “Saving and Wealth Inequality”, CEPR discussion paper 2017 (web)

Gabaix X., “Power Laws in Economics and Finance”, *Annual Review of Economics* 2009 (web)

Garbinti, Bertrand, Jonathan Goupille and Thomas Piketty, “Accounting for Wealth Inequality Dynamics: Methods, Estimates and Simulations for France (1800-2014) ”, working paper, 2017 (web)

Kesten H., “Random difference equations and renewal theory for products of random matrices” *Acta Mathematica* 1973

Piketty, Thomas, *Capital in the 21st Century*, Cambridge: Harvard University Press, 2014 (web)

Piketty, Thomas, and Gabriel Zucman, “Capital is back: wealth-income ratios in rich countries 1700-2010”, *Quarterly Journal of Economics*, 2014 (web)

Piketty, Thomas, and Gabriel Zucman “Wealth and Inheritance in the Long-Run”, Handbook of Income Distribution, 2015 (web)

Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman, “Distributional National Accounts: Methods and Estimates for the United States”, NBER working paper, 2016 (web)

Saez, Emmanuel, and Gabriel Zucman, “Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Returns”, *Quarterly Journal of Economics*, 2016 (web)

Stiglitz J.E. “Distribution of Income and Wealth Amont Individuals”, *Econometrica* 1969 (web)