

# **Econ 133 – Global Inequality and Growth**

## **Models of the wealth distribution**

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## Roadmap

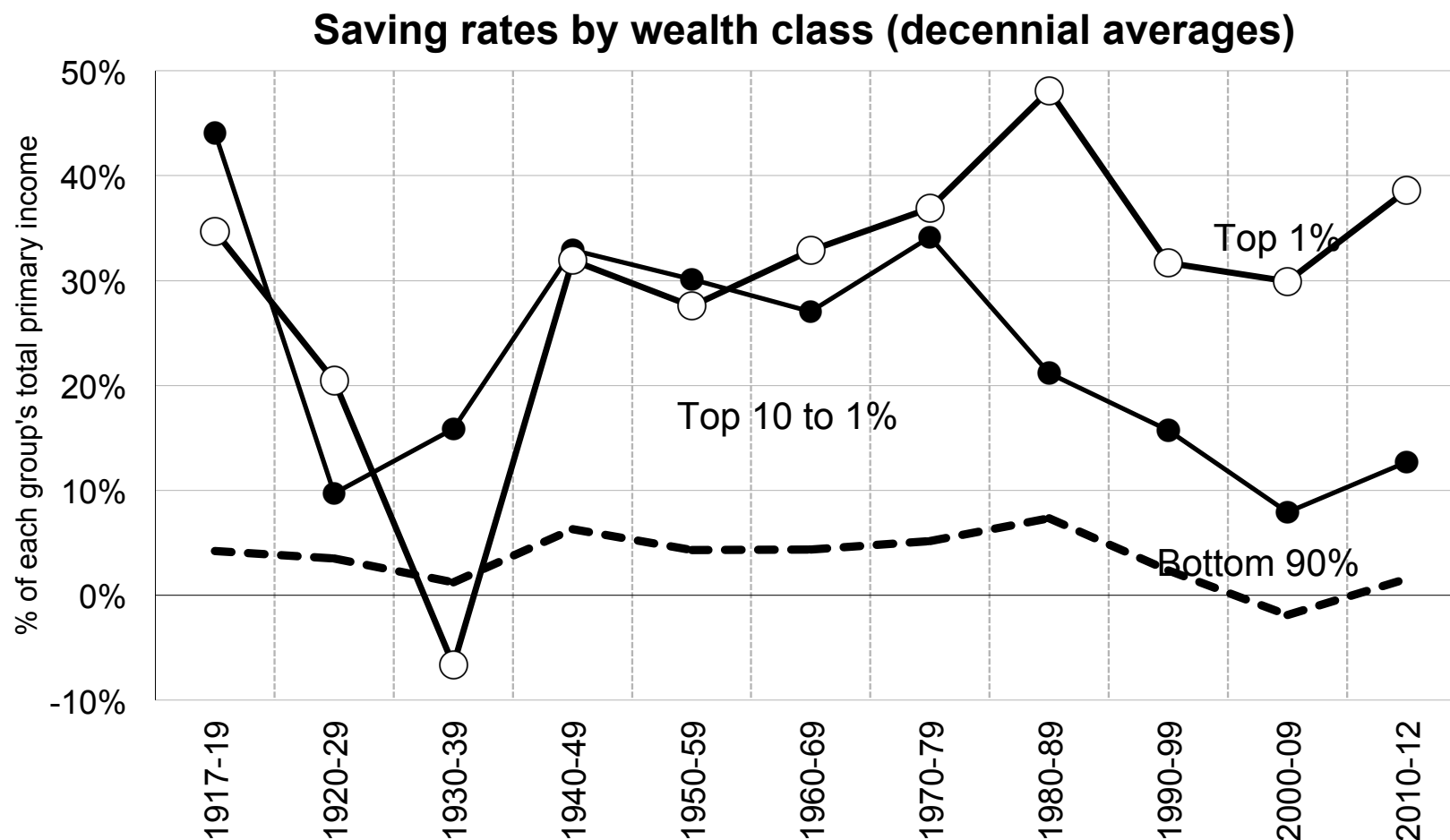
- The precautionary saving model
- The life-cycle model
- Dynamic random shock models

Key question for the study of wealth inequality: why is wealth much more concentrated than labor income?

- Precautionary saving models: wealth less unequally distributed than income
- Life-cycle saving models: wealth as unequally distributed as labor income
- To generate a higher concentration of wealth, one needs dynamic models with cumulative shocks over long horizons

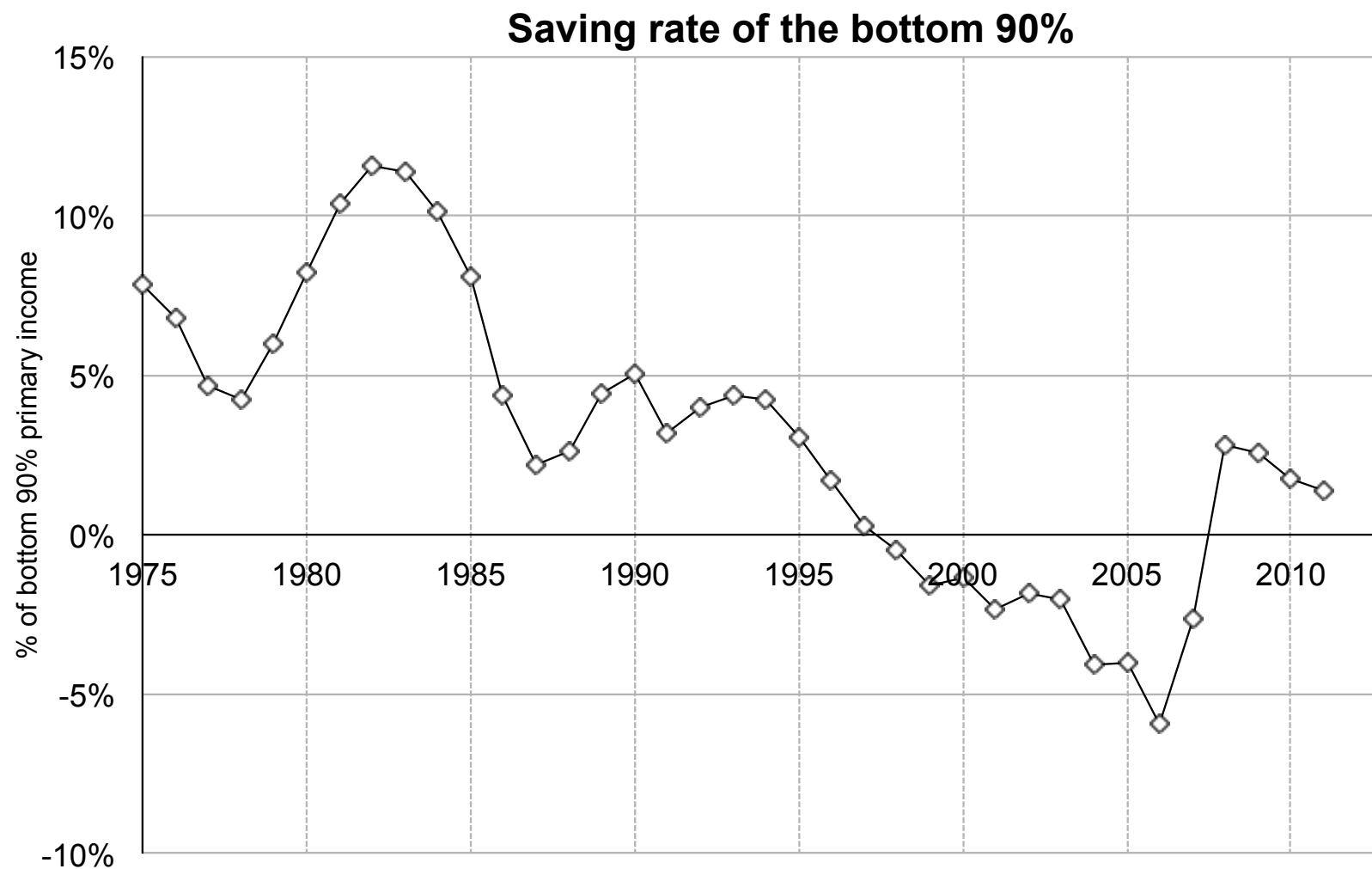
# 1 Precautionary saving model

- Income is uncertain → hold wealth as precaution for “rainy days”
- Main uncertainty: job loss → labor income risk
- As one gets richer, less need to insure against labor income risk → model predicts that saving rate falls with income
- Not consistent with the data



The rich save more as a fraction of their income, except in the 1930s when there was large dis-saving through corporations. NB: The average private saving rate has been 9.8% over 1913-2013.

Source: Saez and Zucman (2016)



Source: Saez and Zucman (2016)

## 2 Life-cycle saving models

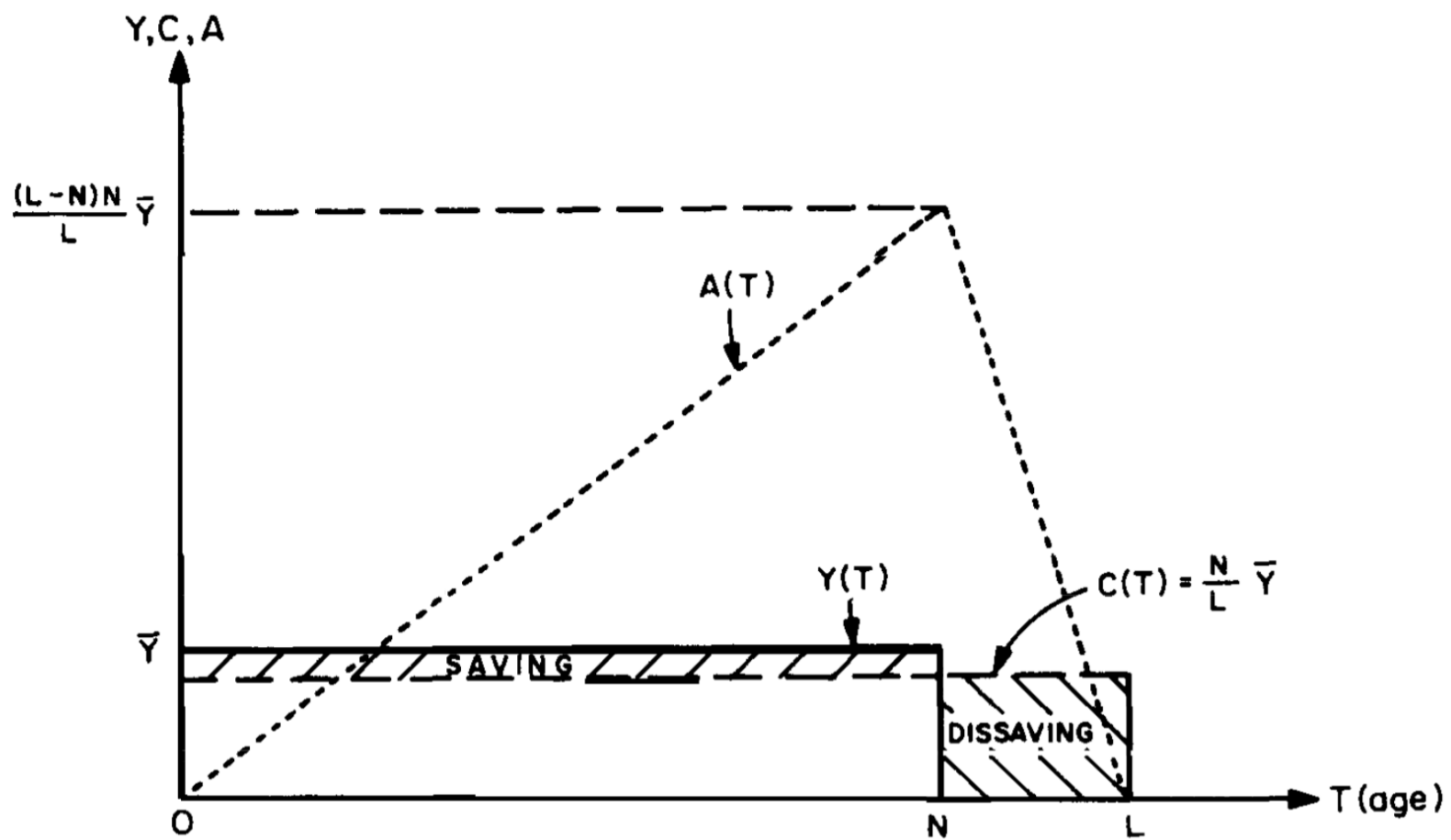
Main idea: people save to spread resources over the life-cycle

### 2.1 A simple life-cycle model

- Individuals die with 0 wealth, wealth accumulation entirely driven by need to save for retirement
- Assume that everybody starts working at age 0, works for  $N$  years, dies at age  $L$ , and that there is no growth ( $n = g = r = 0$ )

- Ex:  $N = 60$ ,  $L = 70 \rightarrow$  retirement length  $L - N = 10$  years
- Labor income is constant at  $\bar{Y}$  during working age period, then 0 during retirement
- Everybody fully smoothes annual consumption so that  $C$  is always equal to average per capita output:  $C = \bar{Y} \cdot N/L$
- While working, people save  $S = (1 - N/L) \cdot \bar{Y}$
- Then during retirement people dis-save  $S = -N/L \cdot \bar{Y}$





INCOME, CONSUMPTION, SAVING AND WEALTH AS A FUNCTION OF AGE

Source: Modigliani (1985)

## 2.2 The Modigliani triangle formula

Aggregate wealth/income ratio = half of retirement length

$$\frac{W}{Y} = \frac{1}{2} \cdot (L - N)$$

Proof:

## 2.3 Predictions of simple life-cycle model

- If retirement length  $L - N = 10$  years, then  $W/Y = 500\% \rightarrow$  model can generate large and reasonable wealth/income ratios
- Aggregate wealth/income ratio is independent of income level and solely depends on demographics
- Model can be extended to  $n > 0, g > 0, r > 0$

Consider an economy with  $n = g = r = 0$ ,  $N = 60$ ,  $L = 70$ , and assume that between age 60 and 70, people work just as much as before 60. Then according to the Modigliani model, the aggregate wealth/income ratio will be:

A — 0%

B — 250%

C — Indeterminate (can take any value)

D — 500%

## 2.4 Limits of simple life-cycle model

- Social Security → reduces need to save for retirement
- What fraction of aggregate wealth comes from life-cycle savers?  
Modigliani vs. Kotlikoff-Summers controversy
- Main limit: life-cycle model generates too little wealth inequality:  
wealth inequality simply the mirror image of income inequality

## **3 Dynamic random shock models**

### **3.1 Different types of shocks**

- Shocks to rates of return
- Shocks to number of children
- Shocks to saving taste across generations

## 3.2 Sketch of a simple dynamic random shock model

Let's consider a model where random shock is a saving taste shock:

- Each period is a generation (30 years)
- Each individual  $i$  receives same labor income  $y_{Lti} = y_{Lt}$  in period  $t$  and has same annual rate of return  $r_{ti} = r_t$
- Each agent chooses  $c_{ti}$  (life-time consumption) and  $w_{t+1i}$  (bequest left to children) so as to maximize a utility function

$$U(c_{ti}, w_{ti}) = c_{ti}^{1-s_{ti}} w_{ti}^{s_{ti}}$$

- where  $s_{ti}$ : bequest taste parameter
- Budget constraint:  $c_{ti} + w_{t+1i} \leq y_{Lt} + (1 + r_t) \cdot w_{ti}$
- Random shocks come from idiosyncratic variations in the saving taste parameter  $s_{ti}$
- $s_{ti}$  drawn from some random process with mean  $s = E(s_{ti}) < 1$



Theorem: under a certain number of assumptions, wealth converges to a steady-state distribution that has the following properties:

- It follows a Pareto law at the top
- The Pareto exponent  $a$  depends on taste shocks  $s_{ti}$
- The higher the variance of shocks, the lower  $a$
- $a \rightarrow 1$  (and thus wealth inequality tends to infinity) if the variance of shocks goes to infinity, and  $a \rightarrow \infty$  if the variance goes to zero

A realistic theory of the wealth distribution has the following property:

A — Wealth inequality rises forever

B — Wealth is more unequally distributed than labor income

C — People mostly save for retirement

D — People mostly save to insure themselves against unemployment risk

## 4 Summary

- There are different saving motives: precautionary, life-cycle, bequest
- Life-cycle and precautionary saving alone cannot explain the level of wealth concentration
- Wealth is very concentrated because of dynamic, random shocks

## References

Modigliani, Franco, “Life Cycle, Individual Thrift and the Wealth of Nations”, Nobel lecture, 1985, (web)

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Saez, Emmanuel, and Gabriel Zucman, “Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Returns”, *Quarterly Journal of Economics*, 2016 (web)