

# Comments on Smith, Zidar and Zwick (2020)

Emmanuel Saez and Gabriel Zucman — May 14<sup>th</sup>, 2020.

Smith, Zidar and Zwick (2020)<sup>3</sup> henceforth SZZ, present estimates of US wealth inequality using the same data (income tax returns) and methodology (income capitalization) as in Saez and Zucman (2016), modifying the benchmark Saez and Zucman (2016) capitalization technique. They write that the rise in the top 0.1% wealth share is “half as large” as in Saez and Zucman (2016). We identify two main flaws in the SZZ methodology. First, SZZ under-estimate the interest-bearing assets of the wealthy because they assume the interest rate earned by the rich is much higher than in the existing evidence. Second, the SZZ methodology under-estimates billionaire equity wealth, because SZZ infer equity wealth based on dividend income despite the fact that the wealthiest Americans often own equities that do not pay dividends. Because of these two biases, the SZZ estimates are inconsistent with the 2016 level and the 2001-2016 rise of top-end US wealth seen in the official Federal Reserve estimates (Survey of Consumer Finances, Distributional Financial Account) and *Forbes*. According to the SCF (which by construction excludes the Forbes 400) appended to the Forbes 400 list, a wealth tax at a rate of 2% above \$50 million and 3% above \$1 billion would have generated \$202 billion in revenue in 2016. According to SZZ the same wealth tax would have raised only \$117 billion. Once the conceptually correct interest rate is used to capitalize interest and the SZZ estimates are fixed to match the estimates of billionaire wealth from *Forbes*, the SZZ estimates are virtually identical to the benchmark Saez and Zucman (2016) series, generate the same amount of wealth tax revenue (namely \$190 billion in 2016), and are reconciled with the Federal Reserve data. In addition to these problems of substance, SZZ do not properly discuss previous work that asked the same question, used the same data, applied similar key assumptions, and obtained similar results.

SZZ modify the benchmark Saez and Zucman (2016) methodology in 2 main ways:

1. First, in contrast to the benchmark Saez and Zucman (2016) series that assume constant interest rates by wealth group, SZZ assume that the rich have a much higher interest rate than the rest of the population. A similar assumption was implemented and investigated in supplementary series constructed in Saez and Zucman (2016, Appendix Tables B40, B41, B41b, and B41c, discussed pp. 549–551) and in Bricker et al. (2018)<sup>4</sup>
2. Second, instead of the equal weights used in the benchmark Saez and Zucman (2016) series, SZZ put a very low weight (10%) on capital gains (vs. 90% on dividends) to estimate equity wealth. A similar assumption was implemented and investigated in supplementary series

---

<sup>3</sup>“Top Wealth in the United States: New Estimates and Implications for Taxing the Rich,” 24 April 2020. [A previous version](#) of these comments, which referred to the [first draft](#) of Smith, Zidar and Zwick (dated July 19, 2019), was circulated in February 2020. Except for fixing a mistake in the computation of wealth tax revenues and slightly amending the interest income capitalization, the responses provided by SZZ (in Appendix J of their [revised draft](#)) do not address the core issues raised in our comments. This note has been expanded and updated to reflect the current SZZ draft (April 24, 2020) and to explain why the SZZ responses do not address our concerns.

<sup>4</sup>Saez and Zucman (2016, Appendix Tables B40, B41, and B41b) construct series where the top 1% by total income earn the 10-year Treasury yield. Bricker et al. (2018, Figure 4) construct series where the top 1% by interest income earn the 10-year Treasury yield. SZZ assume the top 1% by interest income earn the 10-year Treasury yield, as in Bricker et al. (2018, Figure 4), and add the assumption that the top 0.1% earn the Moody’s Aaa corporate bond yield.

constructed in Saez and Zucman (2016, Appendix Tables B36, B37 and B37b discussed pp. 534–535) that put a 0% weight on capital gains (vs. 100% on dividends).

These two changes explain virtually all the difference between the benchmark Saez and Zucman (2016) estimates and the SZZ estimates. Under the benchmark Saez and Zucman (2016) methodology, the share of wealth owned by the top 0.1% richest adults rose from 6.5% in 1978 to 18.6% in 2016 (+12.1 points). Under the modified methodology used by SZZ, the share of wealth owned by the top 0.1% richest adults rose from 6.6% in 1978 to 14.3% in 2016 (+7.7 points). The different capitalization of interest explains the vast majority of the gap.<sup>5</sup>

The alternative assumptions favored by SZZ raise two main problems which had been identified by Saez and Zucman (2016), explaining why these alternative assumptions had not been retained for their benchmark estimates. SZZ do not provide new data or theory to support the case for switching to these alternative assumptions; their implicit justification for applying very high interest rates at the top is conceptually wrong; and their methodological changes lead to large inconsistencies with the other sources about top-end US wealth.

1. First, the high interest rate for the wealthy that SZZ assume is inconsistent with the existing evidence in the datasets where both income and wealth are visible—matched estates-income tax data and the Survey of Consumer Finances (SCF). These data show that the top 0.1% wealthiest Americans have an average interest rate much lower than implied by the SZZ methodology. For instance, in 2016, Americans who died with more than \$20 million in wealth had an interest rate of 1.4%: for any \$1 in interest in their income tax return, they had \$70 in bonds and other interest-bearing assets in their estate. By contrast, in the SZZ methodology Americans with more than \$20 million in wealth are assigned an interest rate of 3.3% in 2016: for any \$1 in interest in their income tax return, SZZ assign them \$30 in interest-bearing assets. As a result, SZZ under-estimate the interest-bearing assets owned by the wealthy.

SZZ respond to this objection by pointing out that although the interest rate they assume is much higher than the interest rate  $r$  of people at the top of the *wealth distribution*, it is similar to the interest rate  $\bar{r}$  of people at the top of the *interest income* distribution. So, which interest rate should conceptually be used to capitalize income when rates of returns are heterogeneous and correlated with wealth,  $r$  or  $\bar{r}$ ? We formally demonstrate below that the answer is  $r$ , and that using  $\bar{r}$ , as SZZ do, generates large biases. The SZZ methodology under-estimates top wealth shares by construction.

2. Second, estimating equity wealth based on dividend income does not allow one to capture top-end wealth accurately, because the wealthiest Americans often own equities that do

---

<sup>5</sup>SZZ implement 3 other changes relative to the benchmark Saez and Zucman (2016) methodology: (i) housing wealth is estimated using state-specific property tax rates (instead of a single property tax rate in Saez and Zucman, 2016); (ii) pensions are estimated using an age-group specific formula combining wages and pension (instead of a single formula irrespective of age in Saez and Zucman, 2016); (iii) pass-through businesses are valued by using formulas by industry of the type:  $\text{wealth} = a \times \text{profits} + b \times \text{sales} + c \times \text{assets}$  (instead of a formula using only profits in Saez and Zucman, 2016). These changes, while useful to provide a more granular picture of wealth, do not materially affect the level of top wealth shares or their trend, since pensions and housing wealth are small at the top, and business assets turn out to be similar in size and distribution in SZZ as in the benchmark Saez and Zucman (2016) series (see point 5.8 below). SZZ also include an estimate of unfunded defined benefit pensions in

not pay dividends. For instance, 5 of the top 10 richest Americans—Jeff Bezos (Amazon), Mark Zuckerberg (Facebook), Warren Buffett (Berkshire Hathaway), Sergei Brin (Alphabet), and Larry Page (Alphabet), collectively worth more than \$250 billion in 2016—were the main shareholders of corporations that did not pay dividends in 2016. The SZZ methodology assigns them a negligible amount of wealth relative to their true wealth. As a result, SZZ under-estimate billionaire wealth by about 40%. In 2016, US billionaires owned 3.0 trillion in wealth according to Forbes, 3.1 trillion in the benchmark Saez and Zucman (2016) series, and 1.7 trillion according to SZZ. The benchmark Saez and Zucman (2016) methodology is consistent with the evidence on billionaire wealth; the SZZ methodology is not.

Because of these two biases, the SZZ estimates are inconsistent with the dynamic of top-end wealth observed since the turn of the 21st century in the official Federal Reserve data on wealth inequality—the Survey of Consumer Finances and the Distributional Financial Accounts. According to the official Survey of Consumer Finances data, the top 1% wealth share rose 6.2 points between 2001 and 2016.<sup>6</sup> According to the official Distributional Financial Accounts, the top 1% wealth share rose 5.2 points over the same period. According to the benchmark Saez and Zucman (2016) estimates, the top 1% wealth share (for adult individuals) rose 5.4 points.<sup>7</sup> By contrast, according to SZZ the top 1% wealth share rose only 1.4 points over the same period. SZZ provide no evidence that the Federal Reserve over-estimates the rise of wealth inequality since 2001.

The SZZ estimates are also inconsistent with the level of top-end wealth observed in the SCF, the only publicly available dataset with micro-data on top-end wealth. As a result SZZ predict wealth tax revenues that are much below those found using the SCF. Anyone can download the public-use 2016 SCF micro-file—which by construction excludes the Forbes 400—append the 2016 Forbes 400 list, and simulate the mechanical revenue from a wealth tax at rate of 2% above 50 million and 3% above 1 billion in 2016, assuming no tax avoidance. The result is \$202 billion. Using the benchmark Saez and Zucman (2016) methodology, the same wealth tax would have generated a similar amount, \$190 billion. According to SZZ however, the same wealth tax would have raised only 117 billion (SZZ, p. 32). SZZ do not provide a logically consistent explanation for this gap.

To summarize, we identify three main issues with SZZ, which the rest of this note discusses in turn:

1. SZZ under-estimate the interest-bearing assets of the wealthy because SZZ assign them too high interest rates.
2. The SZZ methodology, which almost fully ignores capital gains, is not appropriate to capture billionaire wealth and delivers estimates equal to only 57% of *Forbes*.
3. The SZZ methodology fails to capture the level of top-end wealth recorded in the official Survey of Consumer Finances and its 2001–2016 rise. As a result SZZ under-estimate wealth tax revenues by almost 50%.

---

their measure of wealth (in contrast to Saez and Zucman 2016 who only include funded pension), which reduces wealth concentration but does not affect the level of top-end wealth nor wealth tax revenue estimates.

<sup>6</sup><https://www.federalreserve.gov/publications/files/scf17.pdf> box 3 pp. 10-11, Figure B.

<sup>7</sup>Series following the benchmark Saez and Zucman (2016) methodology updated to 2016 are published in

**1. The SZZ methodology under-estimates the interest-bearing assets of the wealthy by construction.** The SZZ methodology assumes that the wealthy earn a much higher interest rate than the rest of the population. This assumption is inconsistent with the evidence where one can observe both income and wealth, which shows that the wealthy in recent years have earned a slightly higher, but not much higher, interest rate than the rest of the population. SZZ defend their methodology by pointing out that top interest-income earners have a much higher interest rate than average, and thus that it is justified to estimate wealth by applying a high interest rate to top-interest income earners. We formally demonstrate below that this argument is incorrect. Capitalizing income using the interest rate of top interest-income earners delivers downwards biased estimates of top wealth shares by construction.

*1.1. Evidence on the interest rate of the wealthy.* Previous research has investigated how the interest rate on interest-bearing assets varies with wealth in the US, using data where both income and wealth can be observed: matched estates-income tax data, and the SCF. In both datasets, the assumption of constant interest rates by wealth group appears reasonable until 2008–2009 but problematic after.

- Saez and Zucman (2016) match estates tax returns filed over the period 1997–2012 with income tax returns the year before death (see Saez and Zucman p. 547–551, in particular Figure V.B). They find interest rates of 3.0% on average over 2001–2008 for estates above \$10 million (close to the aggregate rate of 2.7% on average), but higher-than-average interest rates at the top over 2009–2011 (2.2% for estates above \$20 million vs. 1.4% on aggregate, a difference of a factor of 1.6).
- Bricker, Henriques & Hansen (2018, Table 1, col. 2) find similar patterns in the SCF: interest rates for the top one percent richest households of 3.0% on average for the 2001, 2004, and 2007 SCF (close to the average SCF interest rate of 2.8% over these 3 years), but higher-than-average interest rates at the top in 2010, 2013 and 2016 SCF (2.0% for the top one percent wealthiest households vs. 1.5% on average in the SCF, a difference of a factor of 1.3).

SZZ successfully replicate these findings, update the series to 2016, and do not analyze any new data source on the interest rate of the wealthy in the US.

Noting that an interest rate premium had appeared at the top after the Great Recession, Saez and Zucman (2016) constructed supplementary series showing how the level, trend, and composition of top wealth shares is affected when higher interest rates—the 10-year Treasury yield or the rate seen in matched estates-income tax data—are assumed at the top.<sup>8</sup> Relatedly, Bricker et al. (2018) analyzed how the Saez and Zucman (2016) benchmark estimates are affected when assuming that the top 1% by wealth (or the top 1% by total income, or the top 1% by interest income) earn the 10-year Treasury yield.

Replicating Bricker et al. (2018, section 4.3.2), SZZ assume that the top 1% by interest income earn the 10-year Treasury yield.<sup>9</sup> In addition, departing from earlier work, SZZ capitalize

---

Piketty, Saez and Zucman (2018), [Appendix Tables II \(tables E\)](#), and on <http://WID.world>.

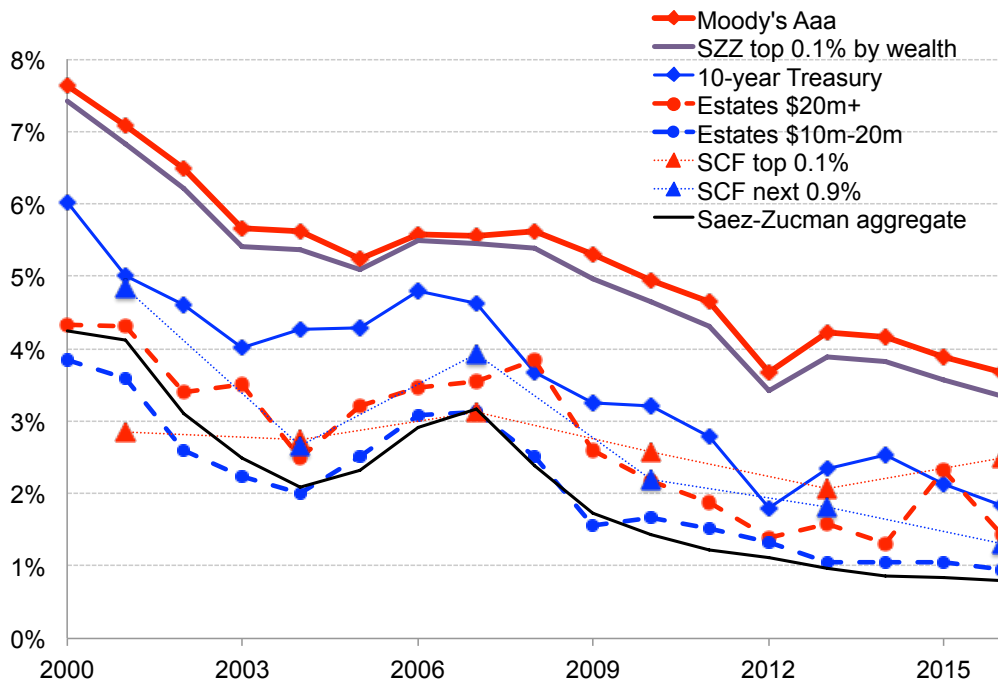
<sup>8</sup>Appendix Tables B40, B41, B41b in Saez and Zucman (2016) assign the 10-year Treasury yield to the top 1% by income; Appendix Table B41c assigns the interest premium seen in matched estates-income tax data to the top 0.1% by wealth; the results are discussed in Saez and Zucman (2016, pp. 549–551).

<sup>9</sup>Saez and Zucman (2016, Appendix Tables B40, B41, and B41b) construct series where the top 1% by total

the interest income of top 0.1% interest-income earners with an even higher rate, the Moody's Aaa yield, which averaged 6.0% over 2000–2009 and 4.2% over 2010–2016.

As shown by the figure below, over the 2001-2016 period, the Moody's Aaa rate (plain red) was always much higher than the interest rate for the top 0.1% richest people observed both in matched estates-income tax data and in the SCF (dashed red), by a factor of 2.0 on average. Similarly, the 10-year Treasury yield (plain blue) was always higher than the interest rate for the next 0.9% richest people observed both in matched estates-income tax data and in the SCF (dashed blue), by a factor of 1.5 on average. The top 0.1% wealthiest Americans in the SZZ methodology (purple line) have an interest rate very close to the Moody's rate, much higher than the real-world top 0.1% wealthiest Americans.

Figure 6: Interest Rates Observed at the Top of the Wealth Distribution vs. Moody's Aaa and 10-Year Treasury Yields



Source: Interest rates in matched estates-income tax returns taken from Saez and Zucman (2016, Appendix Table C6b) for 2000–2011 and from SZZ Appendix Figure A.15 Panel A for 2012–2016. Interest rates in the SCF are from Saez and Zucman (2019, Figure 3).

1.2. *The SZZ methodology delivers biased top wealth shares by construction.* SZZ justify the use of a very high interest rate to capitalize interest income by noting that although the interest rate

income earn the 10-year Treasury yield, while SZZ rank by interest income. Bricker et al. (2018, Figure 4) rank by interest income as SZZ.

<sup>10</sup>SZZ also defend their approach in their appendix J by arguing that it is “more practically useful” to apply heterogeneous returns to bins of interest income, since interest income is observable in the data and wealth is not. But this argument conflates two issues—what is the conceptually correct  $r$  to apply, and how, practically, to implement the capitalization method—and is also incorrect. Conceptually, as we show below, the interest rate to apply is the interest rate of the wealthy. Practically, one can apply this rate to the wealthy by proceeding by iteration. Using  $\bar{r}$  is neither correct conceptually, nor more practically useful than alternatives.



they assume is much higher than the interest rate  $r$  of people at the top of the *wealth distribution*, it is similar to the interest rate  $\bar{r}$  of people at the top of the *interest income* distribution. From there, SZZ implicitly assume that using  $\bar{r}$  is the appropriate way to capitalize interest income when returns are heterogeneous. They never formally demonstrate that capitalizing interest using  $\bar{r}$  leads to unbiased estimates, or even informally discuss why this might be the case, or what the biases with such a method might be.<sup>[10]</sup>

As we now demonstrate, using  $\bar{r}$  to capitalize interest income when interest rates are heterogeneous generates a first-order downward bias in top wealth shares.

To start with, note that the fact that top-interest income earners have a high interest rate  $\bar{r}$  is unsurprising. It's selection: people are in the top 0.1% of the interest income distribution precisely because they have high interest rate. It's also a consequence of measurement errors. In the 2001 SCF, for example, the top 0.1% interest income earners have a 20% interest rate (SZZ Figure A.15 Panel B)—an implausibly high rate due to data inconsistencies, such as misclassification of non-interest income as interest or under-reporting of interest-bearing assets. The high interest rate observed at the top of the interest income distribution is a mechanical consequence of idiosyncratic returns, whether these returns are real or reflect measurement errors.

Let's now prove that estimating the amount of interest-bearing assets by using  $\bar{r}$  generates a large downwards bias in top wealth shares. Saez and Zucman (2016) formally discuss biases in the capitalization method when returns are idiosyncratic (Section IV.A), biases when returns are correlated with wealth (Section IV.B), and how theoretical biases change with multiple asset classes. Building on this earlier work, we provide here a model that allows one to consider these three issues simultaneously (details and proof are in the attached appendix A).

We assume that wealth is Pareto distributed at the top (above fractile  $p_0$ ) with Pareto coefficient  $a$ . The goal is to estimate the share of wealth  $sh_p$  owned by those above fractile  $p$  (where  $p > p_0$ , for example  $p = 0.999$ ). The true wealth of person  $i$  is  $W_i$ . It is split into interest-bearing assets (share  $\beta_i$  of wealth) vs. other assets (share  $1 - \beta_i$ ). To focus on the issue of interest capitalization, we assume that interest rates are heterogeneous while non-interest bearing assets can be estimated perfectly. Let  $r_i$  be the interest rate of person  $i$ . Interest income of person  $i$  is  $r_i\beta_iW_i$ . Let  $r_c$  be the interest rate used to estimate wealth by capitalization. Capitalized interest-bearing assets are  $\beta_iW_i r_i/r_c$  so that wealth estimated by capitalization is  $W_i^c = (1 - \beta_i + \beta_i r_i/r_c) \cdot W_i$ . Under regularity assumptions (namely, that the distributions of  $r_i$  and  $\beta_i$  conditional on wealth eventually converge at the top to uncorrelated distributions with means  $r$  and  $\beta$ ), the wealth share of those above fractile  $p$  estimated by capitalization,  $sh_p^c$ , and their true wealth share,  $sh_p$ , are related by the following formula:

$$sh_p^c = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i/r_c)^a])^{\frac{1}{a}} \quad (1)$$

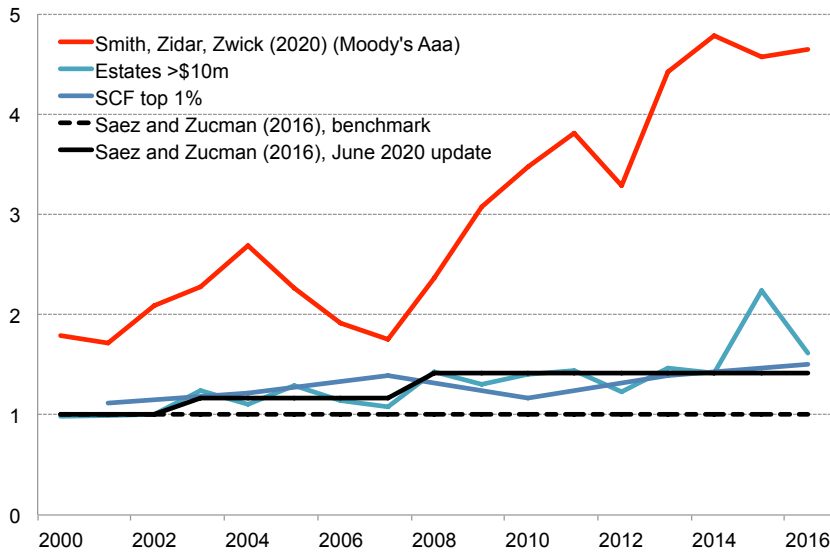
To a first-order approximation, when the share of interest-bearing assets in wealth  $\beta_i$  is small relative to 1, the ratio  $sh_p^c/sh_p$  is equal to  $1 - \beta(1 - r/r_c)$  where  $r$  is the true average interest rate of the wealthy and  $r_c$  is the interest rate used for capitalization. To see this, note that for  $\beta_i$  small,  $(1 - \beta_i + \beta_i r_i/r_c)^a \simeq 1 + a\beta_i(r_i/r_c - 1)$  and hence  $[E[(1 - \beta_i + \beta_i r_i/r_c)^a]]^{1/a} \simeq [1 + a\beta E(r_i/r_c - 1)]^{1/a} \simeq 1 - \beta(1 - r/r_c)$ . When  $a$  is close to one (top heavy wealth), this is an exact formula. Empirically,  $a \simeq 1.4$  (wealth is highly concentrated) so that the approximation is very accurate for  $\beta$  in the empirical range of 0.2–0.4. Two conclusions follow:

1. Zero first-order bias requires using  $r_c = r$ , i.e., requires using the average interest rate of the *wealthy*,  $r$ , to capitalize interest.

- If as done by SZZ, one capitalizes interest using the average interest rate of top  $p$  interest earners,  $\bar{r}$ , there will be a first order bias in the estimated top wealth shares. For example, with  $\beta = 0.25$  and  $r/\bar{r}$  of 0.33, then  $sh_p^c = 0.83 \cdot sh_p$ <sup>11</sup>. Instead of a true top 0.1% wealth share of 18%, the SZZ methodology of capitalizing interest using  $\bar{r}$  delivers a biased estimated top 0.1% wealth share of 15%. We provide more discussion and complete proofs in Appendix A.

To illustrate the biases in the different methods used to capitalize interest, it is useful to plot  $r_c/r_m$ , the interest rate used to capitalize interest at the top, divided by the macro interest rate  $r_m$ . The benchmark Saez and Zucman (2016) series use  $r_c = r_m$ . SZZ use  $r_c = \bar{r}$  the Moody's Aaa rate, a rate which is 4.6 times larger than the average interest rate  $r_m$  in 2016. In both matched estates-income tax data and the SCF, the average interest rate of the wealthy,  $r$ , is close to  $r_m$  before the Great Recession, and averages around 1.4 times  $r_m$  over 2008–2016.<sup>12</sup> This calls for using  $r_c = r = 1.4 \times r_m$  after the Great Recession, i.e., for increasing the interest rate of the wealthy by a factor of around 1.4 compared to the benchmark Saez and Zucman (2016) methodology.<sup>13</sup> SZZ increase the interest rate of the wealthy by a factor of **4.6** in 2016.

Figure 7: Interest rate used for capitalization at the top divided by macro interest rate ( $r_c/r_m$ )



Notes: Estates >\$10m is the weighed average of the interest rate for estates above \$20 million (with a weight of 2) and estates between \$10 million and \$20 million (with a weight of 1) from Figure 1 above, divided by the Saez-Zucman aggregate interest rate from Figure 1 above. SCF top 1% is the top 1% interest rate in the SCF divided by the average interest rate in the SCF.

To sum up: When interest rates are heterogeneous and higher on average among the wealthy, capitalizing interest income using the interest rate of top-interest income earners  $\bar{r}$ , as SZZ do, generates a large downward bias in estimated top wealth shares. Zero first-order bias requires

<sup>11</sup>In 2016,  $r/\bar{r} = 0.3$  in matched estates-income data and 0.5 in the SCF for the top 0.1% (SZZ Figure A.15).

<sup>12</sup>For the top 0.1% in the SCF,  $r/r_m$  is slightly higher (1.57). Top-end interest rates are noisy due to small sample sizes, and interest rates in the SCF are upward biased, probably more so at the very top than on average (biasing  $r/r_m$  upwards), see point 5.5 below.

<sup>13</sup>The June 2020 update of the Saez and Zucman (2016) will use  $r_c = 1.4 \times r_m$  over 2008–2016.

using  $r$ , the interest rate of the wealthy. Capitalizing interest using  $r$  instead of  $\bar{r}$  increases the SZZ top 0.1% wealth share by 2.9 points in 2016, from 14.3% to 17.2%.<sup>14</sup> This single adjustment to the SZZ methodology closes 67% of the gap between SZZ and the benchmark estimates of Saez and Zucman (2016) (top 0.1% wealth share of 18.6% in 2016), while being consistent with the interest-rate differential observed between the wealthy and less wealthy.

**2. The SZZ methodology is not appropriate to capture top-end equity wealth and delivers estimates of billionaire wealth much lower than *Forbes*.** The SZZ methodology under-estimates top-end equity wealth, because SZZ infer equity wealth based primarily on dividend income despite the fact that the wealthiest Americans often own equities that do not pay dividends. As a result, the SZZ methodology generates only 57% of the wealth of billionaires implied by *Forbes*.

*2.1. The SZZ methodology fails to match the evidence on billionaire wealth.* According to SZZ (2020, p. 32) billionaires owned \$1.7 trillion in wealth in 2016. According to *Forbes*, the top 400 wealthiest Americans (who had wealth above \$1.7 billion) owned \$2.4 trillion in 2016. Billionaires with more than 1 billion and less than \$1.7 billion add close to an extra \$600 billion, for a total billionaire wealth of around \$3 trillion.<sup>15</sup> SZZ only capture 57% of that amount.

SZZ under-estimate billionaire wealth because their methodology is not appropriate to estimate equity wealth (in non-pass through businesses), the key form of wealth at the top. SZZ put a very low weight (10%) on capital gains (vs. 90% on dividends) to estimate equity wealth. A similar assumption was implemented and investigated in supplementary series constructed in Saez and Zucman (2016, Appendix Tables B36, B37 and B37b discussed pp. 534–535) that put a 0% weight on capital gains (vs. 100% on dividends) and this assumption was found to under-estimate top end wealth, because many of the wealthiest Americans are major shareholders of companies that do not pay dividends (Amazon, Google, Facebook, Berkshire Hathaway, etc.).

Two examples illustrate the problem with the SZZ methodology. In 2016 the capitalization factor for dividends plus capital gains is 40 in the SZZ methodology, which means that people are assigned 36 times their dividends plus 4 times their capital gains in equity wealth. Someone with 0 dividend gets 4 times his realized capital gains in equity wealth. According to SEC Form 4 public records, in 2016 Jeff Bezos sold around 2 million Amazon stocks at a price of around \$700, resulting in up to 1.4 billion in capital gains. In the SZZ methodology, the implied equity wealth is  $4 \times \$1.4 \text{ billion} = \$5.6 \text{ billion}$ . That same year, Bezos’s stake in Amazon was valued at

---

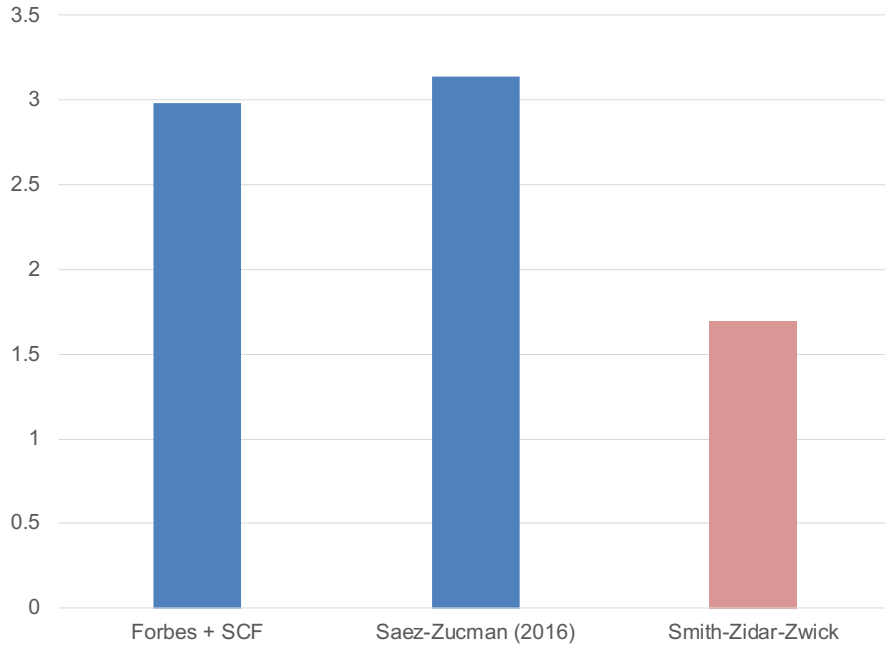
<sup>14</sup>In 2016, the average interest rate of the wealthy in matched estates-income tax data  $r$  is 1.3%, the average rate used by SZZ to capitalize the interest of the top 0.1% is the Moody’s rate  $r_c = 3.7\%$ , so that with  $\beta = 0.25$  there is a bias of  $1 - \beta(1 - r/r_c) = 0.83$ . Starting from the SZZ top 0.1% wealth share of 14.3%, getting rid of the bias by setting  $r_c = r$  increases the estimated top 0.1% wealth share to  $14.3\%/0.83 = 17.2\%$ . Note that the bias generated when using the interest rate of the top interest earners is larger than the bias of opposite sign generated when using the homogeneous macro interest rate  $r_m$ . Over 2008–2016,  $r/r_m$  has averaged around 1.4, so that with  $\beta = 0.25$ , homogeneous capitalization leads to over-estimating top wealth shares by  $1 - \beta(1 - r/r_m) = 1.10$ .

<sup>15</sup>There are two ways to arrive at the \$600 billion number for non-*Forbes* 400 billionaire wealth. First, one can make the classical assumption that the tail of the wealth distribution is Pareto distributed. As the average wealth of the *Forbes* 400 in 2016 (\$6.0b) was 3.5 times the threshold to belong to the *Forbes* 400 (\$1.7b), the corresponding Pareto parameter is  $a = 3.5/(3.5 - 1) = 1.4$ . Standard calculations imply that the wealth between \$1bn and \$1.7bn is  $[(1.7/1)^{(a-1)} - 1] = 23.6\%$  of the wealth above \$1.7bn, i.e., \$567 billion. Second, one can look at SCF data: the public-use 2016 SCF file, which by construction excludes the *Forbes* 400, has \$583 billion in billionaire wealth (with by construction wealth below 1.7 billion).



around \$60 billion. To take another example, in 2016 Warren Buffett [disclosed](#) he had adjusted gross income of \$11,563,931 in 2015. Assuming this all came from capital gains (as Berkshire Hathaway does not pay dividends), the implied wealth in the SZZ model is \$50 million. That year, Buffett’s stake in Berkshire Hathaway was worth about \$60 billion.

Figure 8: Wealth of American billionaires in 2016 (Trillion of \$)



Notes: Forbes + SCF is the estimate of billionaire wealth obtained by appending the Forbes 400 to the SCF, which by construction excludes the Forbes 400. Saez-Zucman (2016) is the estimate from the Distributional National Accounts microfiles of Piketty, Saez and Zucman (2018), which use the benchmark Saez and Zucman (2016) method to estimate wealth. Both the SZZ and the Saez and Zucman (2016) estimates reported here are at the tax unit level, conceptually comparable to the unit of observation for *Forbes* and SCF at the top.

As shown by the graph above, the benchmark Saez and Zucman (2016) capitalization method captures close to 100% of the amount of billionaire wealth implied by *Forbes*. By contrast, the SZZ methodology, which under-estimates top-end equity wealth, under-estimates billionaire wealth by almost 2.

2.2. *SZZ do not provide evidence that the Forbes 400 aggregate is wrong.* In their Section 10.3.2 and appendix J, SZZ instead provide a biased overview of the literature on the reliability of the *Forbes* ranking. They note that some billionaires have over-stated their wealth to *Forbes*—but do not mention that *Forbes* misses some billionaires, since people above the *Forbes* 400 threshold but who do not appear in *Forbes* have been sampled by the SCF (Batty et al., 2019, Appendix F). They note that private businesses are hard to value—but diversified portfolios of stocks and bonds, for which there is no public information, are even harder to capture and likely to be missed by *Forbes*. They refer to the [Bloomberg billionaire index](#) in their footnote 60—but do not mention that the Bloomberg index finds as much and sometimes even more wealth at the top than *Forbes*. Even the small point on Pareto coefficients does not reflect the

current state of knowledge. SZZ claim that the Forbes-400-based Pareto parameter of 1.4 in 2016 implies top 0.1%, top 1%, and top 10% wealth shares that line up more closely with their preferred estimates of top shares than with the Saez and Zucman (2016) benchmark estimates. However, the literature emphasizes that the Pareto approximation is only valid locally and that the Pareto coefficient is not constant from billionaires down to the top 10% threshold; see, e.g., Blanchet, Fournier, Piketty (2017) and references therein.<sup>16</sup>

*Forbes* is certainly not perfect. However, to learn about the wealth of the richest American, the *Forbes* approach of looking at ownership in large businesses to figure out the true wealth of Jeff Bezos, Mark Zuckerberg, Warren Buffett, Sergei Brin, Larry Page, Elon Musk, Michael Dell, etc., is obviously superior to trying to infer wealth from dividends that haven't yet materialized. Saez and Zucman (2016) opted for a method with a higher weight on capital gains precisely because it did a better job at matching *Forbes*. The gap between SZZ and *Forbes* is the sign of an issue in the SZZ methodology.

Correcting the SZZ estimates so that they match the aggregate billionaire wealth implied by *Forbes*, without making any other correction, increases the SZZ top 0.1% wealth share by 1.7 points in 2016, closing 40% of the gap with the benchmark Saez and Zucman (2016) series.

To sum up, fixing two issues in the SZZ series closes 90% of the gap between the SZZ top 0.1% wealth share and the benchmark Saez and Zucman (2016) estimate:

- Using the conceptually correct interest rate to capitalize interest income increases the SZZ top 0.1% wealth share from 14.3% to 17.2% in 2016.
- Matching the amount of billionaire wealth implied by *Forbes* further increases the SZZ top 0.1% wealth share to 18.1%.<sup>17</sup>

These two adjustments close almost 90% of the initial 4.3 points gap with the benchmark Saez and Zucman (2016) estimate of 18.6%. They also bring the SZZ top 0.1% wealth share within less than 5% of the Saez and Zucman (2016) benchmark estimate ( $18.1 / 18.6 = 97\%$ ). In both level and trend, the SZZ top 0.1% wealth share becomes almost identical to the Saez and Zucman (2016) series.

**3. The SZZ estimates are inconsistent with the level of top-end wealth observed in the Survey of Consumer Finances,** the only publicly available dataset with micro-data on top-end wealth. As a result, SZZ predict wealth tax revenues that are much below those found using the SCF. SZZ do not provide a logically consistent explanation for this gap.

*3.1 SZZ find much less wealth in the top 0.1% (and hence potential wealth tax revenue) than existing sources.* Anyone can download the public-use 2016 SCF micro-file—which by construction excludes the Forbes 400—append the 2016 Forbes 400 list, and simulate the mechanical revenue

---

<sup>16</sup>SZZ also claim that “in terms of the number of billionaires and their collective wealth, statistics from the Forbes 400 fall in between our estimates and those from the equal-return approach of Saez and Zucman (2016),” suggesting that the benchmark Saez and Zucman (2016) methodology delivers too high results for billionaire wealth. However, as Figure 3 above shows, the benchmark Saez and Zucman (2016) methodology delivers an estimate of billionaire wealth very similar to the one implied by *Forbes*, \$3.14T in 2016. A close estimate (\$3.27 trillion) can be obtained in the publicly available Distributional National Accounts micro-files of Piketty, Saez and Zucman (2018), which are blurred at the top for confidentiality reasons.

<sup>17</sup>Coming after the interest rate correction, the billionaire correction adds less than 1.7 points (namely, about 1 point), because the interest rate correction already increases billionaire wealth.

from a wealth tax at rate of 2% above 50 million and 3% above 1 billion in 2016, assuming no tax avoidance. The result is \$202 billion. The code is below:

```
* Define path
global root "..."
cd "$root"

* Download and save SCF
copy "https://www.federalreserve.gov/econres/files/scfp2016s.zip" "scf.zip"
unzipfile "scf.zip"
use rscfp2016.dta, clear
keep networkth wgt
save "scf"

* Append Forbes 400
copy "http://gabriel-zucman.eu/files/misc/forbes2016.csv" "forbes2016.csv"
import delimited "forbes2016.csv", delimiter(";") clear
keep networkth
gen wgt = 1
append using "scf"

* Compute revenue of a wealth tax at 2% above 50 million and 3% above 1 billion
gen tax = 0.02 * max(0, networkth - 5e7) + 0.01 * max(0, networkth - 1e9)
sum tax [w = wgt]
local total = round('r(sum)' / 1e9)
di "WEALTH TAX REVENUE IN 2016: 'total' BILLION"
```

WEALTH TAX REVENUE IN 2016: 202 BILLION

The same result can be obtained without running any code using the wealth tax calculator published by [Vox](#), which also uses the SCF appended to the Forbes 400.<sup>18</sup>

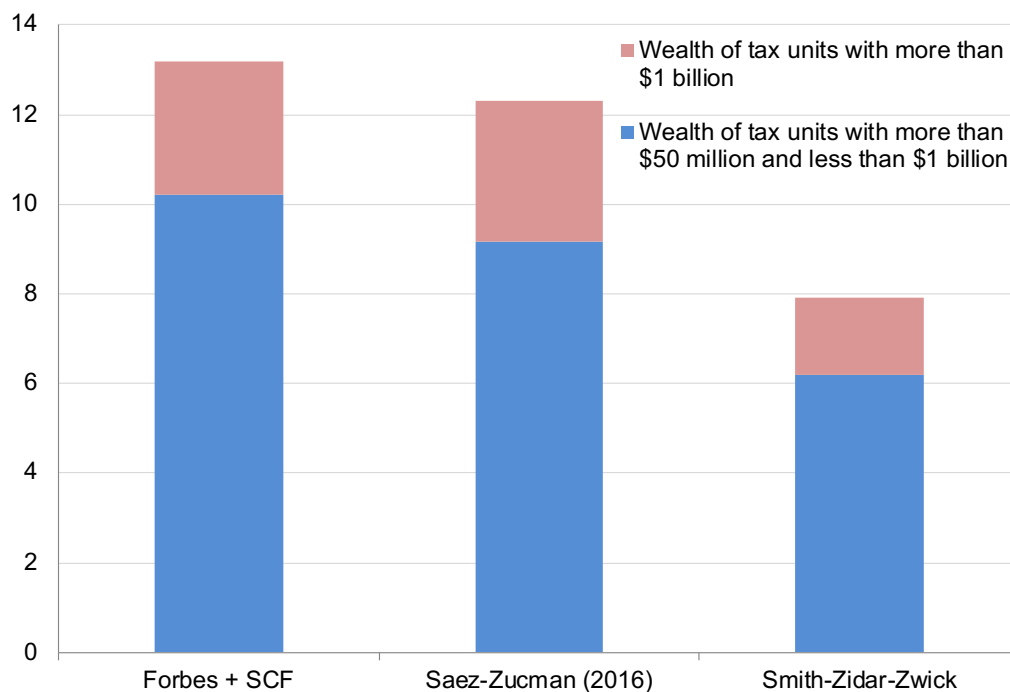
According to the SZZ methodology (SZZ, p.32), the same wealth tax, with the same assumption about avoidance, would have raised only 117 billion in 2016. This is only 58% of the SCF + *Forbes* estimate for the same year. As the figure below shows, this discrepancy is due to the fact that SZZ have both less billionaire wealth than in *Forbes* + SCF (see point 2 above), and less wealth for tax units with wealth between \$50 million and \$1 billion than in the SCF, in both cases by a factor of 1.7. SZZ under-estimate the wealth of tax units with more than \$50 million by a factor of 1.7 compared to existing sources. By contrast, the benchmark Saez and Zucman (2016) methodology is consistent with both the level of billionaire wealth found in *Forbes* + SCF, and the level of wealth for tax units with net wealth between \$50 million and \$1 billion seen in the SCF.

*3.2. SZZ do not provide a logically consistent explanation for the difference in top-end wealth between their estimates and the SCF.* SZZ (p. 42) suggest that the Survey of Consumer Finances

---

<sup>18</sup>The [Vox](#) calculator rounds results to the nearest 10 billion, and so in 2016 it reports that a tax of 2% above \$50 million and 3% above \$1 billion (with no avoidance) would have generated \$200 billion.

Figure 9: Wealth of tax units with net wealth above \$50 million in 2016 (\$ Trillion)



Notes: This figure compares the amount of wealth owned by tax units with more than \$50 million in net wealth in 2016 according to three sources: the Survey of Consumer Finances supplemented by the *Forbes* 400, the benchmark Saez and Zucman (2016) estimates, and SZZ. Saez and Zucman (2016) capture 90% of the wealth of tax units with net wealth between \$50 million and \$1 billion recorded in the SCF, and 105% of the SCF + *Forbes* billionaire wealth. SZZ (2016) capture 61% of the wealth of tax units with net wealth between \$50 million and \$1 billion recorded in the SCF, and 57% of the SCF + *Forbes* billionaire wealth.

over-estimates top-end business wealth.<sup>19</sup> Business assets would have to be over-estimated by a factor of 3 to reconcile the SCF and SZZ estimates of top-end wealth. Instead of owning business assets worth \$6 trillion, families with net wealth above \$50 million (slightly above the top 0.1% threshold, \$43.2 million in 2016) would have to own business assets worth only \$2 trillion in 2016. SZZ do not provide evidence to support the notion that the business assets of top 0.1% families are over-estimated by a factor of 3 in the SCF.

Moreover, this potential reconciliation is logically inconsistent. If the SCF over-estimates the wealth of the top 0.1% by a factor of 1.7 because it over-estimates the business assets of top-end families by a factor of 3, then the portfolio shares observed in the SCF are biased. The shares of all-non-business assets in the portfolios of top 0.1% families must be multiplied by 1.7, while the share of business assets must be divided by  $3 / 1.7 = 1.7$ . The SZZ methodology,

<sup>19</sup>SZZ seem to hesitate between claiming that the SCF over-estimates top-end wealth, and claiming that their results are consistent with the SCF. For instance on p. 42 SZZ write that “Our preferred series closely fits the most comparable equal-split SCF series that makes all adjustments, trending similarly and matching the levels of the top 1% and rising somewhat above the top 0.1% in the 2000s.” This assertion is impossible to understand, given that the top 1% and top 0.1% “most comparable equal-split SCF wealth series” rise as much as the benchmark Saez and Zucman (2016) equal-split series from 2001 to 2016, while the top 1% and top 0.1% SZZ “preferred series” barely increase from 2001 to 2016. See point 5.2 below.

however, is based on fitting the observed portfolio shares at the top-end in the SCF (and other auxiliary moments in the SCF, such as 2001–2016 changes in portfolio shares, the interest rate of the rich, etc.); see, e.g., SZZ Table 3. The argument that business assets are too high in the SCF results in a logical contradiction: if the SCF over-estimates business assets, it does not make sense to fit the portfolio shares observed at the top-end of the SCF<sup>20</sup>

In brief, SZZ attempt to estimate the wealth of the top 0.1%, using the SCF and a methodology whose results imply that the SCF is fundamentally flawed when it comes to estimating the wealth of the top 0.1%. More broadly, SZZ take as gospel auxiliary moments in the SCF that are either mis-measured (such as top-end interest rates, see point 5.5 below) or not comparable across sources (such as portfolio compositions, see point 5.9 below) while discarding the level of wealth found in the SCF—which is what the SCF aims at capturing accurately.

To be sure, the SCF may over-estimate top-end wealth. Using the benchmark Saez and Zucman (2016) methodology, a 2% wealth tax above \$50 million and 3% above \$1 billion would have generated slightly less revenue than implied by the SCF + *Forbes*, 190 billion vs. 202 billion in 2016. The reason is that total net wealth in the SCF exceeds total net wealth in the Financial Accounts, by about 10% (see Batty et al., 2019, Table 1). Saez and Zucman (2019) take a conservative approach to estimate wealth tax revenues and re-scale the SCF so that the SCF + *Forbes* aggregate matches the Financial Accounts total. This reduces the tax base by 10%–15% compared to the raw SCF + *Forbes* tax base. The notion that the SCF over-estimates top-end wealth by a factor of close to 2, however, has no empirical basis.

**4. Improper discussion of previous literature.** SZZ do not adequately discuss the previous literature that asked the same question (how are the benchmark Saez and Zucman, 2016, estimates of US wealth inequality affected by returns heterogeneity?), used the same data, made similar key assumptions, and obtained similar results. SZZ are not the first to investigate the consequences of return heterogeneity for estimates of US wealth inequality based on income tax data—a fact that their current draft fails to make clear.

*4.1. Bricker et al. (2018).* SZZ do not appropriately discuss the closely related work by Bricker et al. (2018). Bricker et al. (2018) use income tax data, the income capitalization method, and capitalize the interest income of the top 1% interest-earners using the 10-year Treasury yield. Like Bricker et al. (2018), SZZ use income tax data, the income capitalization method, and capitalize the interest income of the top 1% interest-earners using the 10-year Treasury yield. Compared to Bricker et al. (2018), SZZ add the assumption that the top 0.1% by interest earn an even higher interest rate, the Moody’s Aaa corporate bond yield. As shown by the Figure below, the SZZ top 0.1% wealth share is close to the Bricker et al. (2018) estimate. The reader of SZZ cannot know this, since although Bricker et al. (2018) are cited by SZZ, their

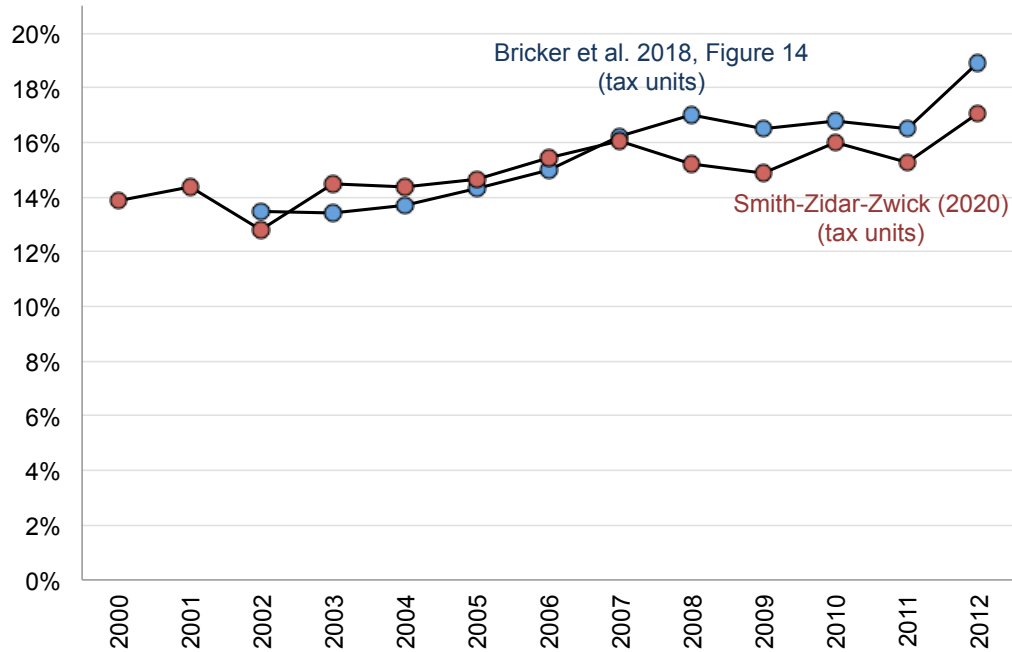
---

<sup>20</sup>Even if the SCF over-estimates business assets and hence has biased portfolio shares, part of the SZZ Table 3 validation exercise could still have merit if the SCF has the correct level of interest-bearing assets at the top. However, what is counted as interest-bearing assets in the SCF vs. in the Financial Accounts is different, so that one should not aim at matching the SCF level of interest-bearing assets as SZZ do. In the Financial Accounts, the interest-bearing assets owned through domestic hedge funds and private equity funds are recorded as interest-bearing assets, whereas they are typically business assets in the SCF (e.g., for the funds’ general partners). This turns out to matter quantitatively, because hedge fund and private equity fund managers play a prominent role at the top of the wealth distribution, and these funds hold large amounts of interest-bearing assets. See point 5.8 below.



methodology and quantitative results are never discussed.<sup>21</sup>

Figure 10: Top 0.1% wealth share: Bricker et al. (2018) vs. SZZ (2020).



Notes: This figure compares the wealth owned by the top 0.1% richest tax units reported in Bricker et al. (2018), Figure 14, obtained by applying the 10-year Treasury yield to interest income for tax units in the top 1% of the wealth distribution, and the top 0.1% wealth share of SZZ among tax units. Both papers use the same data, make similar assumptions and end up with similar results, with the lower level of the top 0.1% wealth share in recent years due to the fact that SZZ apply an even higher interest rate than Bricker et al. (2018) for the top 0.1%.

After quoting Bricker et al. (2018) together with a paper about Norway, SZZ (p. 6) write that “Our contribution is to build on these insights by implementing proposed adjustments in the tax data and combining them with other first-order refinements to all other major asset categories.” However, (i) Bricker et al. (2018) also implemented their insights in the US tax data, and (ii) the “other first-order refinements” of SZZ, whether individually or taken altogether, have a second-order effect on the level, trend, and composition of top wealth shares.<sup>22</sup> 80% of the difference between the SZZ top 0.1% wealth share and the benchmark Saez and Zucman (2016) estimate is due to the different capitalization of interest income, an issue thoroughly investigated in previous work.

<sup>21</sup>SZZ briefly allude to aspects of Bricker et al. (2018) in their footnote 40, failing to note that Bricker et al. (2018) do not only focus on the top 1% but also investigate the implication of heterogeneous returns for the top 0.1% wealth share (see Bricker et al., 2018, figure 14), the main focus of SZZ.

<sup>22</sup>As shown by SZZ Figure 14.B, the SZZ refinements of housing wealth increases top 0.1% wealth by 0.1 trillion in 2016 (0.15% of total wealth), reduces pension wealth by 0.2 trillion (0.3% of total wealth), increases business wealth by 0.4 trillion (0.6% of total wealth, see point 5.8 below), and reduces public equity wealth by 1.0 trillion (1.4% of total wealth), for a total net effect of -0.9% of total wealth. These adjustments are individually and collectively second-order compared to the benchmark Saez and Zucman (2016) top 0.1% wealth share of 18.6% in 2016.

As shown in point 1 above, using the Moody’s Aaa rate to capitalize top 0.1% interest income is conceptually incorrect and generates large biases after the Great Recession. Using the 10-year Treasury yield to capitalize interest for the entire top 1%, as in Bricker et al. (2018), makes the SZZ series virtually identical to Bricker et al.’s (2018, Figure 14) top 0.1% wealth share.<sup>23</sup>

4.2. *Heterogeneous returns series in Saez and Zucman (2016)*. SZZ do not mention that Saez and Zucman (2016) constructed and discussed detailed appendix series showing how the level, trend, and composition of top wealth shares are affected when higher interest rates at the top are assumed. Appendix Tables B40, B41, B41b in Saez and Zucman (2016) assign the 10-year Treasury yield to the top 1% by income. Appendix Table B41c assigns the interest premium seen in matched estates-income tax data to the top 0.1% by wealth. These series are discussed pp. 549-551 of the published Saez and Zucman (2016) paper. The SZZ top 0.1% wealth share tracks the top 0.1% wealth share in Saez and Zucman’s (2016) Appendix Table B41c, see Figure below. SZZ misrepresent our work in their Section 10.1.1 (p. 35) by claiming these series were constructed in subsequent work of ours—they were in fact constructed and discussed in our original paper.<sup>24</sup>

SZZ also claim they “make a methodological contribution by clarifying how capitalization works in practice and by emphasizing both heterogeneity and the concomitant uncertainty that arises.” These issues were discussed in Saez and Zucman (2016), both conceptually and empirically, e.g., in Section III.B.1 titled “How the Capitalization Technique Works,” Section IV titled “Pros and Cons of the Capitalization Method”—including Section IV.A titled “Idiosyncratic Returns,” Section IV.B titled “Returns Correlated with Wealth,” and Section IV.F, e.g., discussion starting with “to assess the quantitative implication of the interest rate differential seen in matched estates-income tax data...” SZZ contribute no new data source or theory to these questions.

We shared all our programs and data infrastructure with SZZ. We welcome replications and extensions of our work and put all our code online for that purpose. It is essential for scientific progress, however, to clearly acknowledge prior work.

Saez and Zucman (2016, p. 550) called for monitoring the evolution of the interest rate differential observed in the post-Great Recession years (as few post-Great Recession years were still available at the time of their study, which ended in 2012, making it hard to assess whether the differential observed in matched estates-income tax data post-2008 reflected statistical noise or a real phenomenon) and to adjust the capitalization method accordingly if need be.<sup>25</sup> The matched estates-income data covering the years 2012–2016 have generally confirmed the presence

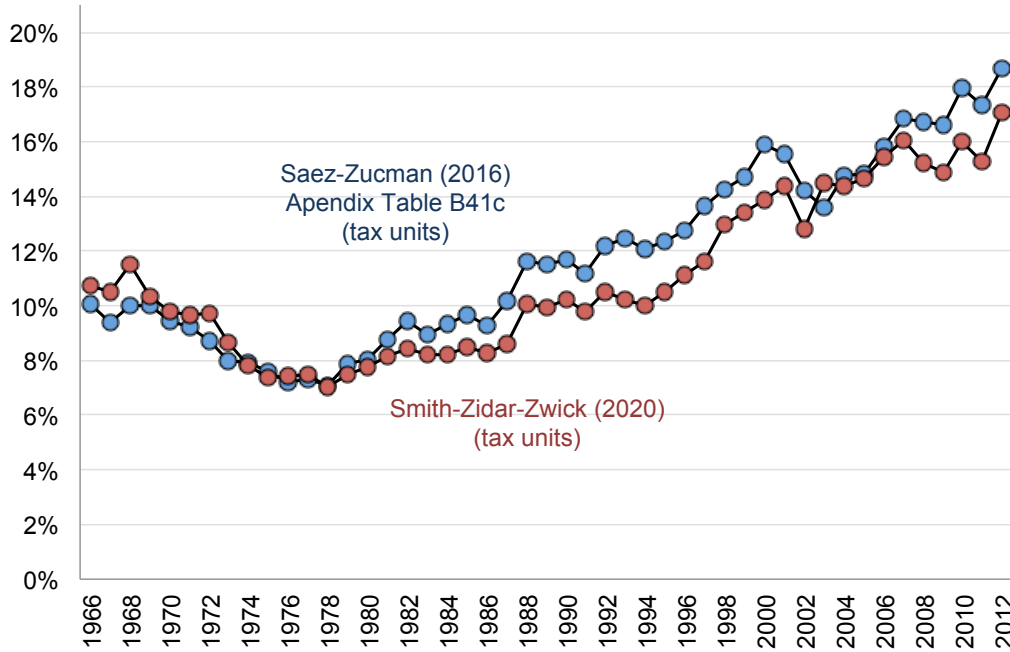
---

<sup>23</sup>One small methodological difference between SZZ and Bricker et al. (2018, Figure 14) is that for the purpose of estimating the top 0.1% wealth share, Bricker et al. (2018) capitalize interest income ranking people by wealth, while SZZ capitalize interest income ranking people by interest income. However, Bricker et al. (2018, Figure 4) show that for a given assumption about the interest rate, ranking by interest income vs. ranking by wealth makes virtually no difference.

<sup>24</sup>SZZ (p. 35) write: “Saez and Zucman (2019a) have subsequently acknowledged this approach introduces bias and make an adjustment to the equal-returns specification by adopting a higher interest rate for their top wealth group. Specifically, they scale down their top fixed income wealth estimate using the interest rate differential from estate taxes matched to income tax returns for estates above \$20 million, which is closer to the 10-year US Treasury rate (UST10) than to the Aaa rate we use,” failing to note that this adjustment was implemented and discussed in our original paper published in 2016 (Appendix B41c series, discussed p. 550).

<sup>25</sup>See Saez and Zucman (2016, p. 550): “We retain our baseline top 0.1% wealth share estimate because only a few hundred non-married individuals die with estates above \$20 million each year. As a result, there is likely

Figure 11: Top 0.1% wealth share: series constructed in Saez and Zucman (2016) by assuming interest rate heterogeneity vs. SZZ (2020) main series



Notes: This figure compares the share of wealth owned by the top 0.1% richest tax units reported in Saez and Zucman (2016), Appendix Table B41c (series discussed in Saez and Zucman, 2016, p. 550) and the top 0.1% wealth share of SZZ among tax units.

of an interest rate premium at the top. Over 2008–2016, the interest rate of estates above \$10 million has been equal to around 1.4–1.5 times the macro interest rate on average, see Figure 2 above<sup>26</sup>. SCF data show similar patterns, with  $r/r_m$  of 1.36 on average for the top 1% over 2007–2016.<sup>27</sup> These new data points call for a more moderate adjustment than the one proposed in the Saez and Zucman (2016) Appendix Table B41c series that used  $r/r_m = 1.6$ .

## 5. Other issues.

- 5.1. SZZ misrepresent their key finding in their abstract. They write: “We find that the top 0.1% share of wealth increased from 7% to 14% from 1978 to 2016. While this rise is

significant noise in the annual series, making it difficult to make a precise and systematic inference of the true interest premium at the top. Looking forward, should new evidence show that taxable returns rise or fall with wealth, then it would become necessary to specifically account for this fact—and similarly when applying the capitalization technique to other countries.”

<sup>26</sup>For the 2012–2016 period, we use the interest rates reported by SZZ in their Figure A.15 Panel A. Young wealthy decedents contribute to these estimates with a high weight, since estate data are weighted by the inverse mortality rate by age and gender to be representative of the entire population—a valuable addition in SZZ compared to Saez and Zucman (2016) who did not weight by the inverse mortality rate; see Saez and Zucman (2016, pp. 549) and SZZ footnote 43. The weighted results happen to be similar to the unweighted results, showing that young wealthy decedents have low returns like elderly decedents.

<sup>27</sup>The interest rates in the top 0.1% in the SCF are slightly higher ( $r/r_m = 1.57$ ), but noisy due to small sample sizes. Moreover, interest rates in the SCF are upward biased and probably more so at the very top than on average (slightly biasing  $r/r_m$  upwards), see point 5.5 below.

half as large as prior estimates...” In fact, the true ratio is not half but 64%. From 1978 to 2016, the benchmark Saez and Zucman (2016) top 0.1% wealth share (for equal-split adults) increases 12.1 points<sup>28</sup> From 1978 to 2016, the SZZ top 0.1% wealth share (for equal-split adults) increases 7.7 points (= 64% of 12.1).<sup>29</sup>

- 5.2 The SZZ methodology does not capture the rise in top wealth shares recorded in both the SCF and the Distributional Financial Accounts since 2001. Since the SZZ top 0.1% wealth share is close to the benchmark Saez and Zucman (2016) series before 2001 and only diverges from the early 2000s on<sup>30</sup> the inconsistency of SZZ with the available evidence for the 2001–2016 period is problematic. According to the official SCF results,<sup>31</sup> the top 1% wealth share has increased 6.2 points between 2001 and 2016 (6.7 points when adding the Forbes 400 to the public-use SCF files). According to the official Federal Reserve Distributional Financial Accounts (based on the SCF, but anchored to the Financial Accounts totals and available at the quarterly frequency instead of every 3 years), the top 1% wealth share has increased 5.2 points from mid-2001 to mid-2016. By contrast, the SZZ top 1% wealth share has increased by 1.4 point only.<sup>32</sup> SZZ do not discuss or provide evidence that the Federal Reserve over-estimates the rise of inequality since 2001. In their Sections (10.3.1 and 10.3.3) comparing their results to the Federal Reserve data, and in their Appendix J, SZZ note that there are a number of conceptual differences between the Federal Reserve series and theirs (e.g., the unit of observation and the treatment of defined benefit pensions). These conceptual differences explain some of the SZZ vs. SCF difference in the level of the top 1% and top 0.1% wealth shares, but they do not explain the large differences in the 2001–2016 rise of these top shares.
- 5.3. SZZ provide an inconsistent treatment of unfunded defined benefit pensions. SZZ include \$1.9 trillion in unfunded defined benefit pension wealth in their series (SZZ, footnote 6 p. 8), 30% of the total amount of unfunded defined benefit pensions recorded in the Financial Accounts (\$6.5 trillion in 2016). However, when they compare their series to the SCF, SZZ add all unfunded defined benefit pensions to the SCF (\$6.5 trillion in 2016). Unfunded pensions have increased since 2001 (from \$2.1 trillion in 2001, 22% of national income, to 6.5 trillion in 2016, 40% of national income), and almost none of this wealth goes the top. As a result, the comparison between the SZZ top wealth shares and the SCF top wealth shares (adjusted to incorporate unfunded pensions) reported in Figure A.16 are biased. The divergence between the SCF and SZZ (SCF top shares rise sharply

---

<sup>28</sup>Series following the benchmark Saez and Zucman (2016) methodology updated to 2016 are published in Piketty, Saez and Zucman (2018), Appendix Tables II (tables E), and on <http://WID.world>.

<sup>29</sup>If one considers the period from 1978 to 2012 (the last year printed in Saez and Zucman, 2016), the ratio is 68%. If one considers the period from 1978 to 2014 (the last year printed in Piketty, Saez and Zucman, 2018), the ratio is 65%. The ratio is never close to 50%.

<sup>30</sup>Between 1978 and 2001, the SZZ top 0.1% wealth share rises 6.9 points vs. 8.0 points in the benchmark Saez and Zucman (2018) series, starting from the same level in 1978.

<sup>31</sup>By official SCF results, we mean the results as published in the Federal Reserve Bulletin, e.g., [here](#) box 3 pp. 10-11, Figure B. These official results are based on the internal-use SCF files, which slightly differ from the public-use files.

<sup>32</sup>Most of the difference between the rise of inequality according to SZZ vs. the Federal Reserve comes from the trend for the top 0.1%, not the next 0.9%. According to the official SCF results, the top 0.1% wealth share has increased 4.0 points between 2001 and 2016 (5.0 points when adding the Forbes 400), vs. 0.9 point in SZZ. The Distributional Financial Accounts do not report statistics for the top 0.1%.

between 2001 and 2016 while the SZZ top shares barely rise) is even more pronounced in actual facts than reported in Figure A.16. SZZ do not provide any justification for including 1.9 trillion in unfunded defined benefit pensions (30% of the Financial Accounts aggregate) in their series while including 100% of the Financial Accounts aggregate in the SCF. In both cases the same source, Sabelhaus and Henriques Volz (2019), is cited. Sabelhaus and Henriques Volz (2019) do not provide estimates for unfunded DB pensions separately from funded DB pensions. The Saez and Zucman (2016) series do not include unfunded defined benefit pensions (for the reasons discussed in Saez and Zucman 2016, p. 525–526).

- 5.4. After fixing the identified biases in the SZZ methodology and adopting a consistent treatment of unfunded defined benefit pensions, the SZZ top 0.1% wealth share series is identical to the benchmark Saez and Zucman (2016) estimate. For instance, starting from the SZZ top 0.1% wealth share of 14.3% in 2016, removing the bias in the interest capitalization adds 2.9 points and matching the evidence on billionaire wealth adds close to an extra point, bringing the SZZ top 0.1% wealth share to 18.1% (see point 2 above). Removing unfunded defined benefit pensions increases the top 0.1% wealth share to 18.6%, identical to the Saez and Zucman (2016) estimate. Alternatively, starting from the Saez and Zucman (2016) level of 18.6% and incorporating 1.9 trillion in unfunded defined benefit pensions as in SZZ, the top 0.1% wealth share falls to 18.1%, identical to the SZZ estimate corrected for interest and billionaires. Whether unfunded pensions are included in wealth is immaterial for the level of top-end wealth and wealth tax revenues. Therefore, after fixing the identified biases in the SZZ methodology, the SZZ series generate the same amount of wealth tax revenue as the benchmark Saez and Zucman (2016) series, and are reconciled with the Federal Reserve data on the level of top-end wealth.
- 5.5. SZZ put too much faith in the top-end interest rates seen in the SCF, which are over-estimated by construction. Interest rates in the SCF are upward biased because what is recorded as interest-bearing assets in the SCF only includes a fraction of interest-generating assets. All the interest-generating assets of pass-through businesses (the bank deposits, notes receivable, bonds, etc., owned by S-corporations and partnerships) generate interest for their individual owners, because interest flows to their 1040, and respondents are asked about interest as reported in their 1040. But these assets are typically counted as business assets in the SCF—not as bank accounts, notes receivable, etc., owned by households. Close to 30% of taxable interest income earned by households flows from pass-through businesses in recent years,<sup>33</sup> meaning that that SCF interest rates are overstated by a factor of 1.4 (and potentially more at the top where pass-through income is

---

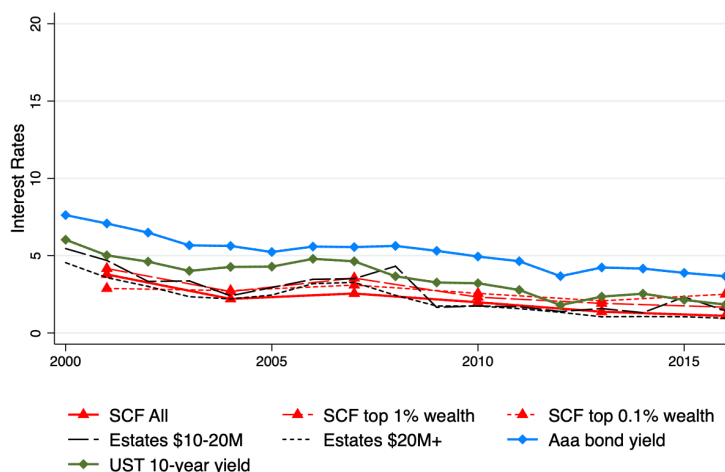
<sup>33</sup>In 2011, Cooper et al. (2016) report that about \$22 billion was paid as taxable interest by partnerships to individuals (around 7% of all partnership income paid to individuals is interest (Figure 8), and about 35% of partnership income is paid to individuals (Figure 3), so out of a total of \$895 billion in partnership income in 2011 (p. 122),  $0.07 \times 0.35 \times \$895 = \$22$  billion was paid as interest to households). In addition, according to IRS statistics, an extra \$11 billion was distributed by S-corporations as interest, for a total of \$33 billion in pass-through interest income, i.e., 28% of the total taxable interest income in 1040s (\$120 billion in 2011). A negligible amount of pass-through interest income was tax-exempt (partnerships and S-corporations owned 80 billion in tax-exempt securities, generating about \$3 billion in interest in 2011). Note also that S-corporations plus partnerships owned about \$3 trillion in interest-bearing assets (with some uncertainty as bonds cannot be separated from equities), and S-corporations had an average interest rate of about 1% on their interest-bearing assets, close to the macro aggregate rate.



prevalent). For instance, instead of 1.67% in 2016, the top 1% interest is closer to 1.2%, consistent with the rate seen in matched estates-income tax data. Note that to some extent the same bias exists in matched estates-income tax data (although less severe), meaning that the interest rate computed in these data should probably be seen as upper bounds.

- 5.6. Illegible key graph. The most valuable empirical finding in SZZ is Figure A.15 Panel A (updating Figure V.B in Saez and Zucman, 2016), showing the interest rate of the wealthy,  $r$ , in the only administrative data source where it can be seen—matched estates-income tax data. However, the graph is illegible, as SZZ increase the y-axis to 20%, despite the fact that the top value in the graph is 7.6% (see graph below). Presumably this is to make the axis consistent with the right panel (Figure A.15 Panel B) that shows the interest rate of top-interest earners  $\bar{r}$ . Inflating the y-axis, however, obscures the large gap between  $r$  and the top-end rates assumed by SZZ. It also illustrates the key problem in the SZZ approach, namely that  $\bar{r}/r$  is very large—which means that using  $\bar{r}$  to capitalize interest generates severely downwards biased top wealth shares (see point 1 and Appendix A).

## A. By wealth groups



Source: SZZ (2020), Figure A.15, Panel A, whose note starts with “Panel A reproduces Figure 1 of Saez and Zucmans (2020)...” (our figure 1 above).

- 5.7. A recurring theme in SZZ (p. 9, p. 20, p. 38, pp. 44–45) is that the Financial Accounts substantially under-estimate the total value of business assets in the United States. SZZ, however, end up with essentially the same value for total business assets. The SZZ preferred estimate for the wealth of sole proprietorships + partnership + S-corporations + private C-corporations is 85% of national income in 2016 (SZZ Figure 6), of which 14% of national income is for private C-corporations (assumed to be 20% of all C-corporations). The conceptually equivalent number in the Financial Accounts is 79% of national income (partnerships plus sole proprietorships: 50%; S-corporations: 20%, private C-corporations: 9% of national income). Should we think of the SZZ estimates as being too low, or are the Financial Accounts totals accurate after all?

- 5.8. Another recurring theme in SZZ is that business assets at the top play a more important role in their series than in the benchmark Saez and Zucman (2016) series (e.g., p. 3: “We find a larger role for private business wealth and a smaller role for fixed income wealth”). In fact, this finding is the mechanical consequence of using an inconsistent definition of business wealth. In their series, SZZ assume that 20% of C-corporation equity wealth corresponds to private equity, and add that amount to business wealth. When they compare their results to the benchmark Saez and Zucman (2016) series, they do not make that adjustment. When 20% of C-corporation equity wealth is added to business wealth in the benchmark Saez and Zucman (2016) series, there is a comparable amount of business wealth in the top 0.1%: 3.9 trillion in 2016 in SZZ vs. 3.5 trillion in equal-returns series (see SZZ Figure 14 Panel B.)
- 5.9. Fuzzy frontier between business assets and fixed-income claims. Following Kopczuk (2015) and Bricker et al. (2016), SZZ puzzle over the fact that the benchmark estimates of Saez and Zucman (2016), anchored to the Financial Accounts totals, have more interest-bearing assets at the top than the SCF. This misses a key difference in asset definition across these 2 sources. In the Financial Accounts, the interest-bearing assets of domestic hedge funds and private equity funds are recorded as interest-bearing assets of households, whereas they are typically business assets in the SCF (e.g., for the funds’ general partners).<sup>34</sup> This turns out to matter quantitatively, because hedge fund and private equity fund managers play a prominent role at the top of the wealth distribution, and these funds hold large amounts of interest-bearing assets. In 2016, according to IRS tabulations of partnership tax returns, financial partnerships owned \$729 billion in cash, \$190 billion in trade notes and accounts receivable, \$150 billion in US government obligations, \$71 billion in mortgage and real estate loans, \$39 billion in tax-exempt bonds, and \$2 trillion in other current assets (listed equities, corporate and foreign bonds, etc.). Assuming one-third of other current assets were interest-bearing, financial partnerships had \$1.8 trillion in interest-bearing assets, of which half with a close to zero interest rate. A considerable fraction of these assets belong to the top 0.1%.<sup>35</sup> An approach based on the Financial Accounts must generate more interest-bearing assets at the top than in the SCF.<sup>36</sup>
- 5.10. SZZ under-estimate pension wealth at the top. SZZ have \$1.9 trillion in pension wealth for the top 1% in 2016. This is lower than in the existing evidence. According to the SCF (which only includes defined contribution pensions) supplemented by the Sabelhaus and Henriques-Volz (2019) estimates of defined benefit pensions, the top 1% wealthiest tax units had \$2.9 trillion in pension wealth in 2016.<sup>37</sup> and the top 1% adults had \$2.6

---

<sup>34</sup>In the SCF, business owners including general partners are asked to estimate the value of their business assets by answering the following question: “What is the net worth of (your share of) this business?”.

<sup>35</sup>Non-profits and foreigners typically invest in offshore hedge funds, which are excluded from these statistics and from the Financial Accounts household balance sheet aggregates. Wealthy US individuals typically invest in onshore funds, which are captured by these statistics.

<sup>36</sup>It is apparent that what gets counted as interest-bearing assets in the SCF vs. the Financial Accounts is different when one compares the SCF vs. Financial Accounts totals. “Time deposits and short-term investments”, by far the largest form of interest-bearing assets and one of the easiest form of wealth to capture in a survey, are consistently twice larger in the Financial Accounts than the conceptually similar category in the SCF (Batty et al. 2019, Table 1). The SCF coverage has deteriorated over time, from 61% on average over 1989–2001 to 47% over 2004–2016 .

<sup>37</sup>Sum of retqliq variable in the SCF (\$2.5T) and the defined benefit estimates of Sabelhaus and Henriques-

trillion (SZZ Figure 14 Panel C).<sup>38</sup> This gap is not due to the fact that SZZ allocate only 1/3 of unfunded defined pensions (see point 5.3 above) since unfunded DB pensions are negligible for the top 1%. The gap is due to another problem is that SZZ approach. SZZ rely on taxable pension distributions to allocate the defined-contribution pension wealth of retirees (see SZZ p. 39), despite the fact that some of the more concentrated forms of defined contribution pensions (e.g., Roth IRAs) do not generate taxable income. To match the level of pension wealth at the top seen in the SCF, it is necessary to allocate part of pension wealth proportionally to non-taxable pension income, as done in Saez and Zucman (2016). One may debate what weight to put on non-taxable pension distributions (vs. taxable distributions), but a 0 weight, as used by SZZ, fails to match the SCF evidence on the distribution of DC pension—the only evidence on the distribution of DC pension wealth in the US.

---

Volz (2019) (\$0.4T) for the top 1% tax units.

<sup>38</sup>SZZ Figure 14 Panel C report a \$1.8 trillion number for the Distributional Financial Accounts, but this number excludes life-insurance reserves, which are included in SZZ's own pension estimates and in the Sabelhaus and Henriques-Volz (2019) estimates of defined benefit pensions. Including life-insurance reserves brings the top 1% pension assets to \$2.3 trillion in the DFA in 2016. Note also that the DFA do not include individual retirement accounts under pension entitlements, while IRAs are part of pension wealth in SZZ and in the SCF. Once one adds IRAs to pension wealth in the Distributional Financial Accounts, the top 1% tax units have around \$3 trillion in pension wealth, similar to the SCF, and much more than SZZ.

## Appendix A: Should One Capitalize Interest with the Rate of Return of the Wealthy or The Rate of Return of Top Interest Earners?

Smith, Zidar, and Zwick (2020) (SZZ) estimate interest-bearing assets (deposits, saving accounts, bonds, etc.) by assigning the Moody’s Aaa yield to top 0.1% interest earners and the 10-year Treasury yield to the next top 0.9% interest earners. They justify this methodology by arguing that in the SCF, the top interest income earners have a rate of return comparable to these rates. Those with high interest income tend to have both a high wealth but also a high rate of return. Therefore, the rate of return of high interest earners is higher than the rate of return of high wealth holders. Should one capitalize interest by applying the rate of return  $r$  of people at the top of the wealth distribution, or the higher rate of return  $\bar{r}$  of people at the top of the interest income distribution? In this note, we build on Saez and Zucman (2016) (SZ) to explain the biases from each method.

SZ discuss in detail issues that arise when the rate of return is not constant in the population due to (1) idiosyncratic returns (uncorrelated with wealth), (2) a higher rate of return among the wealthy, (3) various asset classes. Idiosyncratic returns create an upward bias in the capitalization method. A higher rate of return for the wealthy naturally requires to use for capitalization this higher rate of return (instead of the macro rate of return). Various asset classes may average out errors. SZ consider issues (1), (2), and (3) separately. In this note, we build on the SZ framework to consider issues (1), (2), and (3) simultaneously. We then examine the biases when using the rate of return of the wealthy vs. the rate of return of high interest earners. Using the rate on return of high interest earners generates a downward bias in estimated top wealth shares that is first order in the share of bonds in the wealth of the wealthy. In contrast, using the rate of return of the wealthy generates an upward bias that is only second order in the share of bonds in the wealth of the wealthy. Therefore, using the rate of return of the wealthy is much better than using the rate of return of high interest earners.

**Formal model and assumptions.** In a population of size one, individual  $i$  has wealth  $W_i$ . We assume that  $W_i$  is Pareto distributed above percentile  $p_0$  so that

$$Pr(W_i \geq W) = 1 - F(W) = p_0 \cdot (W_{p_0}/W)^a$$

with  $W_{p_0}$  the wealth threshold at percentile  $p_0$  (think of  $p_0$  as the top 5%). The goal is to estimate the share of wealth  $sh_p$  owned by the top  $p$  percentile (where  $p$  is a percentile above  $p_0$ , for example the top 1% or top 0.1%).

We assume that there are two asset classes: interest-bearing assets and other assets. For short, we call interest-bearing assets simply “bonds”. Individual  $i$ ’s share of bonds in her wealth  $W_i$  is denoted by  $\beta_i$ . To zoom in on the capitalization issue of bonds, we assume that returns on bonds are heterogeneous while other assets can be capitalized perfectly (either because the rate of return is constant for this other asset class or because the value of these other assets is observed perfectly). Let  $r_i$  be the rate of return on bonds of person  $i$ . Bond wealth of person  $i$  is  $\beta_i W_i$  and interest income is  $r_i \beta_i W_i$ .

Denote by  $r$  the average rate of return of the wealthy within percentile  $p$  and by  $\bar{r}$  the rate of return of high interest earners (those in the top  $p$  percentile of the interest income distribution).

---

<sup>39</sup>It would be useful to use systematic administrative data from Scandinavian countries that record both asset values and capital income to analyze this more systematically and more generally test our theoretical predictions.

Empirically, based on matched estates-income tax data and the SCF,  $\bar{r}$  is much higher than  $r$ , around 2-3 times as high in recent years.<sup>39</sup> The SZ vs. SZZ controversy precisely arises because of a large gap between  $r$  and  $\bar{r}$ .

In the general population,  $W_i, \beta_i, r_i$  may be correlated in complex ways. It is natural, however, to assume that the distribution of  $\beta_i$  and  $r_i$  conditional on wealth  $W_i$  converges at the top of the wealth distribution. Let us further assume for tractability that  $\beta_i$  and  $r_i$  are iid within the top  $p$  percentile of the wealth distribution.<sup>40</sup> The key mathematical result needed is the following lemma:

**Lemma:** Suppose  $W_i$  is Pareto distributed with Pareto parameter  $a$  (above percentile  $p_0$ ). Let  $W'_i$  be a transformation of  $W_i$  such that  $W'_i = e_i W_i$  that preserves means ( $EW'_i = EW_i$ ) and such that  $0 \leq e_i \leq \bar{e}$  is bounded and independent of  $W_i$  (within the top  $p_0$  percentile of  $W_i$ ). Then  $W'_i = e_i W_i$  is also Pareto distributed with parameter  $a$  at the top (above  $\bar{e}W_{p_0}$ ) and the top shares of  $W'_i$  and  $W_i$  (above percentile  $p$  of their respective distributions) are related by

$$sh'_p = sh_p \cdot (Ee_i^a)^{\frac{1}{a}}.$$

In words, the top wealth share for  $W'$  is corrected with the power mean of  $e_i$  with coefficient  $a$ .

*Proof.* Let  $F'(W')$  denote the distribution of  $W'_i$ . We have:

$$1 - F'(W) = Pr(e_i W_i \geq W) = \int_{e_i=0}^{\bar{e}} Pr(W_i \geq W/e_i) dE(e_i) = \int_{e_i=0}^{\bar{e}} p_0 \cdot (e_i W_{p_0}/W)^a dE(e_i).$$

where the second equality uses the assumption that  $W_i$  is Pareto distributed and independent of  $e_i$ .<sup>41</sup> Therefore, we have:

$$1 - F'(W) = p_0 \cdot (W_{p_0}/W)^a \cdot \int_{e_i=0}^{\bar{e}} e_i^a dE(e_i) = (1 - F(W)) \cdot Ee_i^a.$$

Therefore  $W'_i$  is also Pareto distributed with the same parameter  $a$  (and this calculation goes through as long as  $W \geq \bar{e}W_{p_0}$ ). Since a Pareto distribution is a power function, any multiplicative disturbance factors out multiplicatively. From this, it follows that the percentile  $p$  of  $W'_i$  and  $W_i$  are related by  $W'_p = (Ee_i^a)^{\frac{1}{a}} \cdot W_p$  and therefore that  $sh'_p = sh_p \cdot (Ee_i^a)^{\frac{1}{a}}$  as stated.<sup>42</sup> QED. □

**Estimated top wealth shares.** Suppose we capitalize interest using  $r$ , the average interest rate of the wealthy. Bond wealth of person  $i$  is  $\beta_i W_i$ , interest income of person  $i$  is  $r_i \beta_i W_i$  which capitalizes into  $r_i \beta_i W_i / r$ . Other wealth  $(1 - \beta_i) W_i$  is observed or perfectly estimated. Hence, estimated wealth for person  $i$  is  $W'_i = (\beta_i r_i / r + 1 - \beta_i) W_i$ . Applying the lemma with  $e_i = \beta_i r_i / r + 1 - \beta_i$ , we obtain:

<sup>40</sup>To obtain formulas, we do not need to assume that  $\beta_i$  is uncorrelated with  $r_i$  but this assumption is needed to sign biases.

<sup>41</sup>Note that as  $e_i$  is bounded above,  $W_i/e_i$  will fall in the top tail  $p_0$  as long as  $W_i/\bar{e} \geq W_{p_0}$ .

<sup>42</sup>Formally,  $1 - F(W_p) = p = 1 - F'(W'_p) = p_0 \cdot (W_{p_0}/W'_p)^a \cdot Ee_i^a = p_0 \cdot ((Ee_i^a)^{\frac{1}{a}} W_{p_0}/W'_p)^a = 1 - F(W'_p/(Ee_i^a)^{\frac{1}{a}})$  so that  $W_p = W'_p/(Ee_i^a)^{\frac{1}{a}}$ .



**Top wealth share using the average interest rate  $r$  of the wealthy:**

$$sh_p^{SZ} = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i/r)^a])^{\frac{1}{a}} \geq sh_p, \text{ with } r = Er_i \quad (2)$$

There is an upward bias because the power mean of  $(1 - \beta_i + \beta_i r_i/r)$  is higher than the straight mean itself equal to 1 (a point made formally by SZ in the special case  $\beta_i \equiv 1$ ).

Suppose instead we capitalize interest using  $\bar{r}$ , the average interest rate of high interest earners. In this case, estimated wealth for person  $i$  is  $W'_i = (\beta_i r_i/\bar{r} + 1 - \beta_i)W_i$ . Applying the lemma with  $e_i = \beta_i r_i/\bar{r} + 1 - \beta_i$ , we obtain:

**Top wealth share using the interest rate  $\bar{r}$  of high interest income earners:**

$$sh_p^{SZZ} = sh_p \cdot (E[(1 - \beta_i + \beta_i r_i/\bar{r})^a])^{\frac{1}{a}}, \text{ with } \bar{r} = \frac{Er_i^a}{Er_i^{a-1}} \geq r \quad (3)$$

Equations (2) and (3) provide a simple way to estimate the relative biases of the two methods. Obviously, if  $r_i$  is homogeneous in the population then  $r_i = \bar{r} = r$  for all  $i$  and either method generates no bias. Theoretically with dispersion,  $\bar{r}$  is the average of  $r_i$  weighted by  $r_i^{a-1}$  that are larger for large  $r_i$  so that  $\bar{r} > r$ .<sup>43</sup>

Empirically,  $\bar{r}$  is two to three times higher than  $r$  for the very simple reason that the highest interest earners are also selected on high  $r_i$ .

The SZ method uses the same  $r$  as the  $r_i$  in the expectation so  $r_i/r$  averages to one. The SZZ method, by contrast, uses a much higher  $\bar{r}$  than the average  $r_i$  so that  $r_i/r$  averages to  $r/\bar{r}$  which is typically below 1/2, creating downward bias. In words, the SZZ method attributes too little bond wealth at the top because it uses too high a rate of return. However, the formulas do not have a straight mean but a power mean with coefficient  $a$ . As  $a > 1$ , the power mean is more than the straight mean, creating an upward bias in both cases.

Take the case of a very thick top tail where the Pareto parameter  $a$  is close to 1. In that case, the power mean boils down to the straight mean and the SZ method has no bias, while the SZZ method has a straightforward downward bias due to an excessive assumed return. Empirically, the wealth distribution has a very thick tail with  $a = 1.4 - 1.5$  in the US.<sup>44</sup>

Take the case of a small bond share (so that  $\beta_i$  averages to  $\beta$  is fairly small relative to 1 and is independent of  $r_i$ ). In that case, the bias in SZ is second order in  $\beta$  while it is first order in SZZ. With  $\beta_i$  small, we have  $(1 - \beta_i + \beta_i r_i/r)^a \simeq 1 + a\beta_i(r_i/r - 1)$  and hence  $[E(1 - \beta_i + \beta_i r_i/r)^a]^{1/a} \simeq (1 + a\beta E(r_i/r - 1))^{1/a} = 1$  while  $(1 - \beta_i + \beta_i r_i/\bar{r})^a \simeq 1 + a\beta_i(r_i/\bar{r} - 1)$  and hence  $[E(1 - \beta_i + \beta_i r_i/\bar{r})^a]^{1/a} \simeq [1 + a\beta E(r_i/\bar{r} - 1)]^{1/a} = 1 - \beta(1 - r/\bar{r})$ . Intuitively, when the share of bonds is small, ranking with or without estimated bonds is pretty much the same, and therefore to a first approximation the SZZ method just creates a straight downwards bias, as it uses too high a return.

---

<sup>43</sup>Proving  $\bar{r} = Er_i^a/Er_i^{a-1}$  goes as follows. Let  $y_p$  be the  $p$  percentile threshold in the distribution of interest income  $y_i = r_i\beta_i W_i$ . Then  $\bar{r} = \int_{r_i\beta_i W_i \geq y_p} r_i\beta_i W_i / \int_{r_i\beta_i W_i \geq y_p} \beta_i W_i = \int_{r_i, \beta_i} r_i\beta_i \int_{W_i \geq y_p/(r_i\beta_i)} W_i dF(W_i) / \int_{r_i, \beta_i} \beta_i \int_{W_i \geq y_p/(r_i\beta_i)} W_i dF(W_i)$  with  $dF(W_i) = p_0 a W_{p_0}^a / W_i^{1+a}$  Paretian and independent of  $r_i$  and  $\beta_i$  in the upper tail. Hence routine computations show that  $\bar{r} = \int_{r_i, \beta_i} r_i^a \beta_i^a / \int_{r_i, \beta_i} r_i^{a-1} \beta_i^a = \int_{r_i} r_i^a / \int_{r_i} r_i^{a-1}$  where the later equality uses the fact that  $\beta_i$  and  $r_i$  are independent at the top (if they are not, one can only use the first equality).

<sup>44</sup>When wealth is very equally distributed, then  $a$  is large and the power mean is close to the max. In that case SZ has an upward bias of  $r_{\max}/r > 1$  while SZZ has an upward bias of  $r_{\max}/\bar{r} > 1$ . Both methods overestimate

**Simple illustrative and calibrated example.** Suppose the individual rate of return  $r_i$  is either  $\bar{r} > 0$  with probability  $\lambda$  or 0 with probability  $1 - \lambda$ . Only a fraction  $\lambda$  of the wealthy has interest income. The average rate is  $r = \lambda\bar{r}$  while the rate on high interest earners is  $\bar{r}$ . Suppose that  $\beta_i = \beta$  is constant. In this case, it is easy to show that:

$$sh_p^{SZ} = sh_p \cdot \left( (1 - \lambda) \cdot (1 - \beta)^a + \lambda \cdot \left[ 1 - \beta + \frac{\beta}{\lambda} \right]^a \right)^{\frac{1}{a}},$$

$$sh_p^{SZZ} = sh_p \cdot ((1 - \lambda) \cdot (1 - \beta)^a + \lambda)^{\frac{1}{a}}.$$

Take  $\lambda = .5$  (i.e.,  $\bar{r} = 2 \cdot r$ ) and  $a = 1.5$ . For  $\beta = 20\%$  (a number close to SZZ preferred estimates), then  $sh_p^{SZ} = 1.01 \cdot sh_p$  (the power mean upper bias is second order only 1%) while  $sh_p^{SZZ} = .902 \cdot sh_p$  so that the bias is almost exactly due to imputing only half of the bond wealth to the wealthy (10% instead of 20%) because of using a rate of return twice too high. For  $\beta = 40\%$ ,  $sh_p^{SZ} = 1.04 \cdot sh_p$  (the power mean upper bias grows four times as large to 4% as it is second order in  $\beta$ ).  $sh_p^{SZZ} = .812 \cdot sh_p$  so that the first order bias (of 20%) continues to largely dominate.

---

top wealth shares because they wrongly impute heterogeneous wealth because of heterogeneous returns. With very low wealth inequality and heterogeneous returns, both methods fail badly (SZ doing even worse than SZZ).

## References

- Batty, Michael, Jesse Bricker, Joseph Briggs, Elizabeth Holmquist, Susan McIntosh, Kevin Moore, Eric Nielsen, Sarah Reber, Molly Shatto, Kamila Sommer, Tom Sweeney, and Alice Henriques Volz.** 2019. “Introducing the Distributional Financial Accounts of the United States,” Finance and Economics Discussion Series 2019-017. Washington: Board of Governors of the Federal Reserve System.
- Blanchet, Thomas, Juliette Fournier, and Thomas Piketty.** 2017. “Generalized Pareto Curves: Theory and Applications,” WID.world working paper 2017/3.
- Bricker, Jesse, Alice Henriques, Jacob Krimmel, and John Sabelhaus.** 2016. “Measuring Income and Wealth at the Top Using Administrative and Survey Data.” *Brookings Papers on Economic Activity*, Spring 2016, p. 312.
- Bricker, Jesse, Alice Henriques, and Peter Hansen.** 2018. “How Much has Wealth Concentration Grown in the United States? A Re-Examination of Data from 2001-2013.” Finance and Economics Discussion Series 2018-024. Board of Governors of the Federal Reserve System.
- Cooper, Michael, John McClelland, James Pearce, Richard Prinsziano, Joseph Sullivan, Danny Yagan, Owen Zidar, and Eric Zwick.** 2016. “Business in the United States: Who owns it, and how much tax do they pay?” *Tax Policy and the Economy* 30, no. 1: 91-128.
- Kopczuk, Wojciech.** 2015. “What Do We Know about the Evolution of Top Wealth Shares in the United States?” *Journal of Economic Perspectives*, 29(1), 47–66.
- Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman.** 2018. “Distributional National Accounts: Methods and Estimates for the United States.” *Quarterly Journal of Economics* 133, no. 1: 553–609.
- Saez, Emmanuel and Gabriel Zucman.** 2016. “Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data.” *Quarterly Journal of Economics* 131, no. 2: 519-578. Updated series (post-2012) published at <http://gabriel-zucman.eu/usdina/>, Appendix Table II, Table series E and on <http://WID.world>
- Saez, Emmanuel and Gabriel Zucman.** 2019. “Progressive Wealth Taxation.” *Brookings Papers on Economic Activity*, forthcoming.
- Smith, Matthew, Owen Zidar, and Eric Zwick.** 2019. “Top Wealth in America: New Estimates and Implications for Taxing the Rich.” Working Paper.